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NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS IN MAPLE USING THE RUNGE-KUTTA METHOD

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Abstract. Students studying in higher education are taught in differential equations, higher mathematics and differential equations. A mathematical model of a number of processes occurring in nature is brought to the differential equation. Many of the quantities found in nature have their own law. Finding these laws directly is a much more complicated matter. Finding the connection between the quantity being considered, its rate of change and its acceleration is by nature much lighter. As a mathematical representation of this connection, however, ordinary differential equations are formed. In finding a quick and accurate solution to such equations, it is important and significant to use modern computer programs. This article addressed the issue of physics in the Maple program and obtained the result.
Keywords: Maple program, first order linear ordinary differential equation, numerical methods, analytical solution, Bernoulli's equation, function, general integral, approximate methods.

Introduction

In higher education, students are taught differential equations and in higher mathematics differential equations. Because the applications of differential equations are very wide, and not only physical processes, but also important issues of mechanics, chemistry, biology, astronomy, Economics, Management Theory are expressed and researched using differential equations. The study of differential equations consists of three goals: to construct a differential equation representing an event or process; to find an exact or approximate solution to the found equation; analysis and conclusion of the found Solution [1]. Currently, great attention is paid to working with information technology,processing information with modern technical means and its analysis in the educational plan of training bachelors in the technical direction of higher educational institutions, the implementation of Numerical Methods in solving practical issues [2]. It is important that students are able to use modern applications of axbotot technologies to quickly and accurately find solutions to differential equations using such opportunities. Today, there are Maple, MathCad, Matlab, Wolframalpha packages designed to solve many computer-aided issues at high speed, notably mathematical ones.

Maple is a software package, a computer graphics system (more precisely, computer mathematics). He founded the company Waterloo Maple Inc., which has been developing software products since 1984. (Visual) is a development product designed for mathematical calculations, data visualization and modeling. Despite the fact that the maple system has a number of tools designed for numerically solving differential equations and finding integrals, it is designed for symbolic calculations. It has advanced graphical tools. You don't have a government programming language called Pascal.

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Solving elementary or boundary value problems is a very broad term that includes both exact analytical methods and approximate numerical methods. We are familiar with analytical methods from the science of differential equations. These methods allow solving only a narrow class of equations. In particular, these methods are widely used in solving second-order linear differential equations with constant coefficients. Such equations have found application in the study of many physical processes, for example, in the theory of vibrations, in the dynamics of solids, etc. Approximate methods were developed long before the advent of computers. Even now, many of them have not lost their practical significance. Approximate methods are usually divided into two groups: approximate analytical methods (search for an approximate solution to an initial or boundary value problem in a given section in the form of a function); numerical or grid methods (construction of an approximate solution to an initial or boundary value problem in a given section). Modern computing technologies and accumulated experience in the field of computing technology make it possible to approximate large and complex problems of differential equations. The most important aspect of numerical calculations is to obtain the desired approximate solution with sufficient accuracy. Important aspects of this precision are the accuracy of the EM usage, the prevention of errors that may be made in the input data, and errors that occur due to rounding. Today, many modern mathematical packages allow solving ordinary differential equations both analytically and numerically with sufficient accuracy. To do this, it is necessary to familiarize yourself with the computational methods of approximate solution of ordinary differential equations and their properties. At the same time, there are also tasks that need to be solved not by existing methods, but by their modification, a new method and algorithm. In general, the boundary value problem given by an ordinary differential equation: has a unique solution; has no solution; it can have several or infinitely many solutions.

Runge is the Kutta method

Let us be given the problem of finding the first order Ordinary Differential Equation $y' = f(x, y)$ with step h with Runge-Kutta method the approximate value of the satisfiable solution $y(x_0) = y_0$ initial conditions in Section $[x_0, b]$. Dividing a given $[x_0, b]$ cut into n equal pieces by $h = \frac{b - x_0}{n}$ steps, we find the values of the function in the values $x_i = x_{i-1} + h$, i=0,1,2,3,…,n of x. $y_{i+1} = y_i + \Delta y_i$, i=0,1,2,3,…,n. Where $\Delta y_i = \left(\frac{K_1^{(i)} + 2K_2^{(i)} + 2K_3^{(i)} + K_4^{(i)}}{6} \right)$.

The Runge-Kutta coefficients here are defined as follows:

$K_1^{(i)} = hf(x_i, y_i)$,

$K_2^{(i)} = hf(x_i + \frac{h}{2}, y_i + \frac{K_1^{(i)}}{2})$,

$K_3^{(i)} = hf(x_i + \frac{h}{2}, y_i + \frac{K_2^{(i)}}{2})$,

$K_4^{(i)} = hf(x_i + h, y_i + K_3^{(i)})$. [4]

Example .Calculate Equation $y' = x^2 + xy + y^2 + 1$ with step h=0,1 in the Runge-Kutta method of satisfying the initial condition x_0 =0, y_0 =0 defined in [0;1] intervals. Solution:we divide the range [0;1] into $n = \frac{b - x_0}{h} = \frac{1-0}{0.1} = 10$, i.e. 10. $x_1 = x_0 + h = 0 + 0.1 = 0.1$; $x_2 = x_1 + h = 0.1 + 0.1 = 0.2$; $x_3 = 0.3$; $x_4 = 0.4$; $x_5 = 0.5$; $x_6 = 0.6$; $x_7 = 0.7$; $x_8 = 0.8$; $x_9 = 0.9$; $x_{10} = 1.0$.

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The program for calculating the solution values of this equation directly in the Maple program is as follows:

Now we solve the found differential equation in the Maple program.

Maple programm:

```
> restart;

> restart;f:=(x,y)->x^2+x*y(x)+y(x)^2+1;
```

$$f := (x, y) \to x^2 + x\,y(x) + y(x)^2 + 1$$

```
> dsoll:=diff(y(x),x)=f(x,y);
```

$$dsoll := \frac{d}{dx}\,y(x) = x^2 + x\,y(x) + y(x)^2 + 1$$

```
> init1:=y(0)=0;
```

$$init1 := y(0) = 0$$

```
>ans2:=dsolve({dsoll,init1},numeric,method=rkf45);
```

$$ans2 := \textbf{proc}(x\_rkf45)\ \ldots\ \textbf{end proc}$$

```
> ns2:=dsolve({dsoll,init1},numeric,method=classical[heunform],output=array([0.1,0.2,0.3,
0.4,0.5,0.6,0.7,0.8,0.9,1.0]),stepsize=0.1);
```

$ans2 :=$

x	$y(x)$
0.1	0.101500000000000021
0.2	0.209160002203515048
0.3	0.329939890792907920
0.4	0.472593801501335953
0.5	0.649293143846819242
0.6	0.878604945898420196
0.7	1.19161431674882866
0.8	1.64608013515918384
0.9	2.36428242649709120
1.0	3.65666061606488624

Conclusions

Looking at the results above, we can say that in the Runge-Kutta method we can get an exact solution at any point in the cut. To do this, it is enough to give a starting condition. In the dsolve command, the type=numeric command is defined to find the numerical solution to the differential equation (Cauchy problem or boundary issue). Then the command to solve the differential equation is in the form of dsolve(eq, vars, type=numeric, options), where eq-equations, vars - a list of unknown functions, options - parameters that allow you to specify the method of numerical integralization of parameters. Procopts are options used to define an ODE system using a procedure (procedure, start, start, number, and procvars). output-defines the desired result from Dsolve. Stepsize-used to kittize the step size. In the Maple program, the following methods are implemented: method=Rkf45 - Runge-Kutta-Felberg method 4-5-order (set by default); method=dverk78 – Runge-Kutta method 7-8 – order; method=classical-3-order classical Runge-Kutta method; method=gear and method=mgear are single-stage and multi-stage Gear methods.

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