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DIVISIBILITY CRITERIA FOR SOME PRIME NUMBERS

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Abstract: In this paper, we have considered the proofs of the numbers 7, 13, 17, 19, 23, 29 and examples of the division signs
Key words: Number theory, last digit, remainder, integer numbers.

Introduction. Divisibility criteria provide a quick way to check if a number is divisible by another without performing full division. Here, we outline the divisibility rules for numbers 7, 13, 17, 19, 23, and 29, along with proofs based on modular arithmetic. Here are some well-regarded books that discuss divisibility criteria and related concepts in number theory for instance [1]-[4] book provides a solid introduction to number theory, including divisibility rules, modular arithmetic, and proofs for divisibility criteria. It's highly readable and suited for both beginners and advanced students. Another [5] classic text, this book covers divisibility rules, properties of integers, and applications of number theory. It's a good choice for those interested in deeper mathematical theory.

Main part 1. Divisibility by 7

Rule:

To check if a number N is divisible by 7:

1. Double the last digit of N.
2. Subtract this result from the rest of the number.
3. If the result is divisible by 7, then N is also divisible by 7.

Proof:

Let $N = 10a + b$, where a is the number formed by all digits except the last one, and b is the last digit. Since $10 \equiv 3(mod 7)$, we have:

$$10a + b \equiv 3a + b(mod 7).$$

To offset the effect of $3a$, we calculate $a - 2b$.

$$3a + b \equiv 3a + b - 7b(\text{mod} 7)$$
$$3a + b \equiv 3a - 6b(\text{mod} 7)$$

If $a - 2b$ is divisible by 7, then □ is divisible by 7.

Example:

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For $N = 266$:

- Last digit: 6, doubled to get 12.
- Remaining number: 26−12=14, which is divisible by 7. Thus, 266 is divisible by 7.

2. Divisibility by 13

Rule:

To check if a number N is divisible by 13:

1. Multiply the last digit by 9.
2. Subtract this to the remaining part of the number.
3. If the result is divisible by 13, then N is also divisible by 13.

Proof:

Given $N = 10a + b$, where a and b are defined as above. Since $10 \equiv -3(mod 13)$, we can write:

$$10a + b \equiv -3a + b(mod 13).$$
$$-3a + b \equiv -3a + b + 26b(\text{mod}13)$$
$$-3a + b \equiv -3(a - 9b)(\text{mod}13).$$

If $a - 9b$ is divisible by 13, then N is divisible by 13.

Example:

For $N = 5954$:

- Last digit: $4 \times 9 = 36$.
- $595 - 36 = 559 \Rightarrow 55 - 9 \cdot 9 = -26$, which is divisible by 13. Thus, 5954 is divisible by 13.

3. Divisibility by 17

Rule:

To check if N is divisible by 17:

1. Multiply the last digit by 5.
2. Subtract this from the remaining part of the number.
3. If the result is divisible by 17, then N is also divisible by 17.

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Proof:

For $N = 10a + b$, since $10 \equiv -7(mod17)$, we rewrite:

$$10a + b \equiv -7a + b(mod17).$$

Subtracting 5 times the last digit from the rest of the number:

$$a - 5b.$$

If $a - 5b$ is divisible by 17, so is N.

Example:

For $N = 133518$:

- Last digit: $8 \times 5 = 40$.
- Difference:

$13351 - 40 = 13311 \Rightarrow 1331 - 1 \times 5 = 1326 \Rightarrow 132 - 6 \times 5 = 102$, which is divisible by 17. Thus, 133518 is divisible by 17.

4. Divisibility by 19

Rule:

To check if a number N is divisible by 19:

1. Multiply the last digit by 2.
2. Add this to the rest of the number.
3. If the result is divisible by 19, then N is also divisible by 19.

Proof:

Given $N = 10a + b$, and since $10 \equiv -9(mod19)$, we adjust by adding twice the last digit:

$$a + 2b.$$

If $a + 2b$ is divisible by 19, then N is divisible by 19.

Example:

For $N = 92568$:

- Last digit: $8 \times 2 = 16$.

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- Sum: $9256+16=9272 \Rightarrow 927+2\times 2=931 \Rightarrow 93+1\times 2=95$, which is divisible by 19. Thus, 92568 is divisible by 19.

5. Divisibility by 23

Rule:

To check if a number N is divisible by 23:

1. Multiply the last digit by 7.
2. Add this to the remaining part of the number.
3. If the result is divisible by 23, then N is also divisible by 23.

Proof:

For $N=10a+b$ with $10\equiv -13(mod\,23)$ $10a+b\equiv -13a+b(\text{mod}\,23)$, we counteract with:

$$a+7b$$

i.e $-13a+b\equiv -13a+b-4\times 23b(\text{mod}\,23)$ $-13a+b\equiv -13(a+7b)(\text{mod}\,23)$.

If $a+7b$ is divisible by 23, then N is also divisible by 23.

Example:

For $N=1794$:

- Last digit: $4\times 7=28$.
- Sum: $179+28=207$ $20+7\times 7=69$, which is divisible by 23. Thus, 1794 is divisible by 23.

6. Divisibility by 29

Rule:

To check if a number N is divisible by 29:

1. Multiply the last digit by 3.
2. Add this to the rest of the number.
3. If the result is divisible by 29, then N is also divisible by 29.

Proof:

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With $N = 10a + b$ and $10 \equiv -19(mod\,29) \Rightarrow 10a + b \equiv -19a + b(\text{mod}\,29)$, we simplify with:

$$a + 3b.$$

If $a + 3b$ is divisible by 29, then N is also divisible by 29.

Example:

For $N = 174$:

- Last digit: $4 \times 3 = 12$.
- Sum: $17 + 12 = 29$, which is divisible by 29. Thus, 174 is divisible by 29.

These divisibility rules rely on modular arithmetic properties to make calculations more efficient. Each rule allows for quick verification by focusing on the manipulation of the last digit, making these methods especially useful for mental math and simplifying divisibility testing in various applications.

In conclusion, schools usually teach division symbols for 2, 3(9), 4, 5, 11 without remainder, but division symbols for 7, 13, 17, 19, 23, 29 are not considered. We think that the above information will be useful for school teachers and those interested in number theory.

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