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SECOND TYPE INTEGRAL CONDITIONAL PROBLEM FOR A LOADED MIXED PARABOLIC EQUATION

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Abstract. In this article, the one-valued solution of the integral conditional problem for the mixed parabolic equation with perpendicular time directions is studied.
Key words. Mixed parabolic equation, mixed parabolic equation, integral condition, Riman-Liuvill differential operator.

D through $y=0$, $y=h$ and $x=-T$ Let's define a half-strip bounded by straight lines, here $h=const>0$, $T=const>0$. Consider the following loaded equation in this field:

$$0=Lu\equiv\begin{cases}L_1u\equiv u_{xx}-u_y+\lambda\dfrac{d}{dy}u(0,y)+\mu D_{0y}^{-\alpha}u(0,y), & (x,y)\in D_1=D\cap(x>0),\\ L_2u\equiv u_{yy}+u_x, & (x,y)\in D_2=D\cap(x<0),\end{cases}$$

here λ, μ – given real numbers, D_{0y}^{α} – Riman-Liuvill meaning α is a fractional differential operator,

$$D_{0y}^{\alpha}f(y)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dy}\int_0^y(y-t)^{-\alpha}f(t)dt,\ 0<\alpha<1.$$

$Lu=0$ – D is a mixed parabolic equation in the field, D_1 right parabolic in the sphere, D_2 and in the sphere it is inversely parabolic.

Recently, researchers have been conducting research for mixed parabolic equations including fractional order differential operators. Including, [1] [5] if the Jevre problem for the second-order mixed parabolic equation was studied in the work, [2][6] in the work, the problems for the fourth-order mixed parabolic equation were studied by the method of spectral analysis.

In this article, the one-valued solution of the one-integral conditional problem for the loaded mixed parabolic equation with perpendicular time directions is studied.

I **matter.** D defined, continuous and limited in the field $u(x,y)$ funksiya topilsinki, it is D_1 and D_2 areas respectively $L_1u=0$ va $L_2u=0$ satisfy the equations and the following conditions:

$$\lim_{x\to-0}u_x(x,y)=\lim_{x\to+0}u_x(x,y),\qquad 0<y<h;$$

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(1) $$u(x,0)=\varphi_1(x), \qquad 0\le x<+\infty;$$
(2) $$u(x,0)=\varphi_2(x), \quad -T\le x\le 0;$$
(3) $$u(x,h)=a(x)\int_0^h u(x,y)dy+\varphi_3(x), \ -T\le x\le 0,$$

here $a(x)$ and $\varphi_j(x),\ j=\overline{1,3}$ - are the given functions, $\varphi_1(0)=\varphi_2(0)$; $\varphi_1(x)\in C[0,+\infty)$ and limited; $a(x),\ \varphi_2(x),\ \varphi_3(x)\in C[-T,0]$.

We prove that the solution to the given problem exists and is unique. Let's assume, $u(x,y)$ - I be the solution to the problem. Based on the terms of the matter,

(4) $$u(-0,y)=u(+0,y)=\tau(y), \qquad 0\le y\le h;$$
(5) $$\lim_{x\to -0}u_x(x,y)=\lim_{x\to +0}u_x(x,y)=\nu(y), \quad 0<y<h$$

let's introduce the definitions.

It is known, $L_1u=0$ of the equation D_1 determined, continuous, limited and (1) va $\lim_{x\to+0}u_x(x,y)=\nu(y),\ 0\le y\le h$ the solution satisfying the conditions is defined as follows [7]:

$$u(x,y)=\int_0^{+\infty}G(x;\xi,y)\varphi_1(\xi)d\xi-\int_0^y\nu(\eta)G(x,0,y-\eta)d\eta+$$

(6) $$-\int_0^y\int_0^{+\infty}\left[\lambda\tau'(\eta)+\mu D_{0\eta}^{\alpha}\tau(\eta)\right]G(x,\xi,y-\eta)d\xi d\eta,$$

here

$$G(x,\xi,y)=\frac{1}{2\sqrt{\pi y}}\left\{\exp\left[-\frac{(x-\xi)^2}{4y}\right]+\exp\left[-\frac{(x+\xi)^2}{4y}\right]\right\}.$$

(6) in the formula $x\to+0$ we go to the limit. In that case

$$\int_0^{+\infty}G(0;\xi,y-\eta)d\xi=1$$

taking into account the equality, we have the following:

$$\tau(y)=-\frac{1}{\sqrt{\pi}}\int_0^y\nu(\eta)(y-\eta)^{-1/2}d\eta+\int_0^{+\infty}G(0;\xi,y)\varphi_1(\xi)d\xi-$$

(7) $$-\int_0^y\left[\lambda\tau'(\eta)+\mu D_{o\eta}^{-\alpha}\tau(\eta)\right]d\eta.$$

(7) equality from the point of view of integral operator of fractional order [7] using and $\tau(0)=\varphi_1(0)$ taking into account that, we write as follows:

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(8) $$(1+\lambda)\tau(y)=-D_{0y}^{-1/2}\nu(y)-\int_0^y \mu D_{0\eta}^{-\alpha}\tau(\eta)d\eta+F(y),$$

here $F(y)=\lambda\varphi_1(0)+\int_0^{+\infty} G(0,\xi,y)\varphi_1(\xi)d\xi$.

(8) to both sides of the equation $D_{0y}^{1/2}$ by applying the differential operator and $D_{0y}^{1/2}D_{0y}^{-1/2}g(y)=g(y)$ if we consider the formula [7], we have this equality:

(9) $$\nu(y)=-(1+\lambda)D_{0y}^{1/2}\tau(y)-\mu D_{0y}^{1/2}\left\{\int_0^y D_{0\eta}^{-\alpha}\tau(\eta)d\eta\right\}+D_{0y}^{1/2}F(y)$$

.

Using expansions of integral and differential operators of fractional order [7], (9) we simplify the second addendum on the right side of the equation. Using the expansion of the integral operator of fractional order and changing the order of integration in the multiple integral, we get this result:

$$\int_0^y D_{o\eta}^{-\alpha}\tau(\eta)d\eta=\frac{1}{\Gamma(\alpha)}\int_0^y\int_0^\eta(\eta-z)^{\alpha-1}\tau(z)dzd\eta=$$

$$=\frac{1}{\Gamma(\alpha)}\int_0^y\tau(z)dz\int_z^y(\eta-z)^{\alpha-1}d\eta=\frac{1}{\Gamma(1+\alpha)}\int_0^y(y-z)^\alpha\tau(z)dz$$

Taking into account this equality and using the expansion of a fractional differential operator, we get the following equality:

$$D_{0y}^{1/2}\left\{\int_0^y D_{0\eta}^{-\alpha}\tau(\eta)d\eta\right\}=\frac{1}{\sqrt{\pi}\Gamma(1+\alpha)}\frac{d}{dy}\int_0^y(y-z)^{-1/2}\left\{\int_0^z(z-\eta)^\alpha\tau(\eta)d\eta\right\}dz.$$

Changing the order of integration in the multiple integral, using the properties of Euler's beta and gamma functions, and performing some calculations, we arrive at this result:

$$D_{0y}^{\frac{1}{2}}\left\{\int_0^y D_{0\eta}^{-\alpha}\tau(\eta)d\eta\right\}=\Gamma^{-1}\left(\alpha+\frac{1}{2}\right)\int_0^y(y-\eta)^{\alpha-\frac{1}{2}}\tau(\eta)d\eta.$$

Considering this equality, (9) takes the following form:

(10) $$\nu(y)=-(1+\lambda)D_{0y}^{1/2}\tau(y)-\mu\Gamma^{-1}\left(\alpha+\frac{1}{2}\right)\int_0^y(y-\eta)^{\alpha-\frac{1}{2}}\tau(\eta)d\eta+D_{0y}^{1/2}F(y).$$

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(10) – unknown $\tau(y)$ va $\nu(y)$ between functions D_1 is a functional relationship derived from the field.Now $L_2 u = 0$ equation and (3), (4) in the conditions, we tend to zero x. The result is this

(11) $$\tau''(y) + \nu(y) = 0, \qquad 0 < y < h$$

To the equation and the following

(12) $$\tau(0) = \varphi_1(0), \qquad \tau(h) = a(0)\int_0^h \tau(y)dy$$

We will have conditions.

(10) and (11) from the equations $\nu(y)$ out the, $\tau(y)$ the following integro-differential equation is formed with respect to:

$$\tau''(y) - \frac{1+\lambda}{\sqrt{\pi}}\frac{d}{dy}\int_0^y (y-\eta)^{-1/2}\tau(\eta)d\eta -$$

(13) $$-\mu\Gamma^{-1}(\alpha + 1/2)\int_0^y (y-\eta)^{\alpha-\frac{1}{2}}\tau(\eta)d\eta = -D_{0y}^{1/2}F(y), \qquad 0 < y < h.$$

And so, $\tau(y)$ to determine the unknown (13) of the equation (12) we came to the problem of finding a solution that satisfies the conditions. If from this issue $\tau(y)$ if we define single-valued, $\nu(y)$ function (10) is defined by equality. And the solution to the problem D_1 by formula (6) in the field, D_2 and in the field $u_{tt} + u_x = 0$ is defined as the solution of the first boundary value problem for Eq. Therefore, from now on, we will deal with the problem of finding a solution of equation (13) that satisfies the conditions (12).

(13) the equation $[0, y]$ by integrating twice in a row over the interval, $\tau'(0) = C$ after entering the markup and some calculations

$$\tau(y) - \frac{1+\lambda}{\Gamma(3/2)}\int_0^y (y-t)^{1/2}\tau(t)dt -$$

(14) $$-\mu\Gamma^{-1}(\alpha + 5/2)\int_0^y (y-t)^{\alpha+3/2}\tau(t)dt = F_1(y), \; 0 < y < h$$

we get an integral equation of the form, here

$$F_1(y) = \int_0^y\int_0^z D_{0t}^{1/2}F(t)dtdz + \varphi_1(0) + Cy.$$

The following lemma holds:

1-lemma. If $f(y) \in L_1[0,h]$ if, then

(15) $$\tau(y) - \frac{\lambda_1}{\Gamma(\alpha_1)}\int_0^y (y-t)^{\alpha_1-1}\tau(t)dt - \frac{\mu_1}{\Gamma(\beta_1)}\int_0^y (y-t)^{\beta_1-1}\tau(t)dt = f(y)$$

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The solution of the integral equation exists and is unique, and it is defined by the following equation:

(16) $$(\tau(y) = f(y) + \int_0^y R(y,s;\lambda_1,\mu_1) f(s)ds,$$

here α_1, β_1 – positive real numbers, $\lambda_1, \mu_1 \in R$;

$$R(y,s;\lambda_1,\mu_1) = \sum_{i=1}^{+\infty} \mu_1^i (y-s)^{i\beta_1 - 1} E^i_{\alpha_1, i\beta_1}\left[\lambda_1 (y-s)^{\alpha_1}\right],$$

$E^{\rho}_{\alpha_1,\beta_1}(z)$ - is the Prabhakar function [8] , $E^{\rho}_{\alpha_1,\beta_1}(z) = \sum_{n=0}^{+\infty} \frac{(\rho)_n z^n}{\Gamma(\alpha_1 n + \beta_1) n!}$.

Proof. We write equation (16) in the following form:

(17) $$\tau(y) - \frac{\lambda_1}{\Gamma(\alpha_1)} \int_0^y (y-t)^{\alpha_1 - 1} \tau(t)dt = \Phi(y), \quad 0 < y < h,$$

here $$\Phi(y) = \frac{\mu_1}{\Gamma(\beta_1)} \int_0^y (y-t)^{\beta_1 - 1} \tau(t)dt + f(y).$$

If we consider the right side of the equation (17) as a function known in time, then its solution

(18) $$\tau(y) = \Phi(y) + \lambda_1 \int_0^y (y-t)^{\alpha_1 - 1} E_{\alpha_1,\alpha_1}\left[\lambda_1 (y-t)^{\alpha_1}\right] \Phi(t)dt$$

is determined by the formula here $E_{\alpha_1,\beta_1}(z) = \sum_{n=0}^{+\infty} \frac{z^n}{\Gamma(\alpha_1 n + \beta_1)}$ – the two-parameter Mittag-Leffler function [9].

$\Phi(y)$ If we put the expression of the function in (18), we get the following equation:

$$\tau(y) - \frac{\mu_1}{\Gamma(\beta_1)} \int_0^y (y-t)^{\beta_1 - 1} \tau(t)dt -$$

$$-\lambda_1 \int_0^y (y-t)^{\alpha_1 - 1} E_{\alpha_1,\alpha_1}\left[\lambda_1 (y-t)^{\alpha_1}\right] \times \left\{ \frac{\mu_1}{\Gamma(\beta_1)} \int_0^t (t-s)^{\beta_1 - 1} \tau(s)ds + f(t) \right\} dt = f(y).$$

If we change the order of integration in the multiple integral, the last equation takes the following form: $$\tau(y) - \frac{\mu_1}{\Gamma(\beta_1)} \int_0^y (y-s)^{\beta_1 - 1} \tau(s)ds -$$

$$-\frac{\lambda_1 \mu_1}{\Gamma(\beta_1)} \int_0^y \tau(s)ds \int_s^y (y-t)^{\alpha_1 - 1} E_{\alpha_1,\alpha_1}\left[\lambda_1 (y-t)^{\alpha_1}\right] (t-s)^{\beta_1 - 1} dt =$$

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(19) $$=\lambda_1\int_0^y (y-t)^{\alpha_1-1} E_{\alpha_1,\alpha_1}\left[\lambda_1 (y-t)^{\alpha_1}\right] f(t)dt + f(y).$$

In the inner integral $t-s=\eta$ we will do the plastering. As a result

$$\int_s^y (y-t)^{\alpha_1-1} E_{\alpha_1,\alpha_1}\left[\lambda_1 (y-t)^{\alpha_1}\right](t-s)^{\beta_1-1}dt =$$

$$=\int_0^{y-s} (y-s-\eta)^{\alpha_1-1} E_{\alpha_1,\alpha_1}\left[\lambda_1 (y-s-\eta)^{\alpha_1}\right]\eta^{\beta_1-1}d\eta$$

we will have equality. This is from here

$$\frac{1}{\Gamma(\nu)}\int_0^z (z-t) E_{\alpha_1,\beta_1}\left(\lambda_1 t^{\alpha_1}\right) t^{\beta_1-1}dt = z^{\beta_1+\nu-1} E_{\alpha_1,\beta_1+\nu}\left(\lambda z^{\alpha_1}\right)$$

using the formula [9], we get the following result:

$$\frac{1}{\Gamma(\beta_1)}\int_s^y (y-t)^{\alpha_1-1} E_{\alpha_1,\alpha_1}\left[\lambda_1 (y-t)^{\alpha_1}\right](t-s)^{\beta_1-1}dt = (y-s)^{\alpha_1+\beta_1-1} E_{\alpha_1,\alpha_1+\beta_1}\left[\lambda_1 (y-s)^{\alpha_1}\right]$$

.

Then (17) takes the following form:

$$\tau(y)-\frac{\mu_1}{\Gamma(\beta_1)}\int_0^y (y-s)^{\beta_1-1}\tau(s)ds -$$

(20) $$-\lambda_1\mu_1\int_0^y (y-s)^{\alpha_1+\beta_1-1} E_{\alpha_1,\alpha_1+\beta_1}\left[\lambda_1 (y-s)^{\alpha_1}\right]\tau(s)ds = \Phi_1(y),$$

here

$$\Phi_1(y)=\lambda_1\int_0^y (y-t)^{\alpha_1-1} E_{\alpha_1,\alpha_1}\left[\lambda_1 (y-t)^{\alpha_1}\right] f(t)dt + f(y).$$

Using the linear expression of the Mittag-Leffler function,

$$\frac{1}{\Gamma(\beta_1)}(y-s)^{\beta_1-1} + \lambda_1 (y-s)^{\alpha_1+\beta_1-1} E_{\alpha_1,\alpha_1+\beta_1}\left[\lambda_1 (y-s)^{\alpha_1}\right] =$$

$$=\frac{1}{\Gamma(\beta_1)}(y-s)^{\beta_1-1} + \lambda_1 (y-s)^{\alpha_1+\beta_1-1}\sum_{k=0}^{+\infty}\frac{\lambda_1^k (y-s)^{\alpha_1 k}}{\Gamma(\alpha_1 k+\alpha_1+\beta_1)} =$$

$$=(y-s)^{\beta_1-1}\left[\frac{1}{\Gamma(\beta_1)} + \sum_{k=0}^{+\infty}\frac{\lambda_1^{k+1} (y-s)^{\alpha_1 k+\alpha_1}}{\Gamma(\alpha_1 k+\alpha_1+\beta_1)}\right] = (y-s)^{\beta_1-1} E_{\alpha_1,\beta_1}\left[\lambda_1 (y-s)^{\alpha_1}\right]$$

.

Taking this into account, we can write equation (20) as follows:

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(21) $$\tau(y)-\mu_1\int_0^y K(y,s)\tau(s)ds=\Phi_1(y),$$

here

$$K(y,s)=(y-s)^{\beta_1-1}E_{\alpha_1,\beta_1}\left[\lambda_1(y-s)^{\alpha_1}\right].$$

(21) – $\tau(y)$ the second kind of integral equation with respect to the unknown function is the Volterra integral equation, which is equivalent to the integral equation (15). We find the solution of equation (21) using the method of successive approximation.

We calculate the iterated kernels according to the following formulas:

$$K_i(y,s)=\int_s^y K(y,t)K_{i-1}(t,s)dt,\ i=2,3,... \ .$$

$K_2(y,s)$ we calculate:

$$K_2(y,s)=\int_s^y (y-t)^{\beta-1}E_{\alpha_1,\beta_1}\left[\lambda_1(y-t)^{\alpha_1}\right](t-s)^{\beta_1-1}E_{\alpha_1,\beta_1}\left[\lambda_1(t-s)^{\alpha_1}\right]dt\,.$$

[8] from Theorem 5 in the work $\rho=\rho'=1,\ \beta'=\beta_1$ using , it can be shown that

$$K_2(y,s)=\int_s^y (y-t)^{\beta_1-1}E_{\alpha_1,\beta_1}\left[\lambda_1(y-t)^{\alpha_1}\right](t-s)^{\beta_1-1}E_{\alpha_1,\beta_1}\left[\lambda_1(t-s)^{\alpha_1}\right]dt=$$
$$=(y-s)^{2\beta_1-1}E^2_{\alpha_1,2\beta_1}\left[\lambda(y-s)^{\alpha_1}\right].$$

As above, it can also be shown that the following equality holds:

$$K_3(y,s)=\int_s^y (y-t)^{\beta_1-1}E^1_{\alpha_1,\beta_1}\left[\lambda_1(y-t)^{\alpha_1}\right](t-s)^{2\beta_1-1}E^2_{\alpha_1,2\beta_1}\left[\lambda_1(t-s)^{\alpha_1}\right]dt=$$
$$=(y-s)^{3\beta_1-1}E^3_{\alpha_1,3\beta_1}\left[\lambda_1(y-s)^{\alpha_1}\right].$$

Continuing this process, using the method of mathematical induction, we derive the following formula for iterated kernels:

$$K_i(y,s)=(y-s)^{i\beta_1-1}E^i_{\alpha_1,i\beta_1}\left[\lambda_1(y-s)^{\alpha_1}\right].$$

Considering this, the solution of equation (21) is using the resolvent

$$\tau(y)=\Phi_1(y)+\int_0^y R(y,s;\lambda_1,\mu_1)\Phi_1(s)ds,$$

here

$$R(y,s;\lambda_1,\mu_1)=\sum_{i=1}^{+\infty}\mu_1^i(y-s)^{i\beta_1-1}E^i_{\alpha_1,i\beta_1}\left[\lambda_1(y-s)^{\alpha_1}\right].$$

1- the lemma is proved.

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1- lemma from the result $\alpha_1 = 3/2$, $\beta_1 = \alpha + 5/2$, $\lambda_1 = 1 + \lambda$, $\mu_1 = \mu$ was without (14) we find the solution of the equation as follows;

(22) $$\tau(y) = \Phi_1(y) + \int_0^y R(y,s;1+\lambda,\mu)\Phi_1(s)\,ds,$$

here

$$\Phi_1(y) = f_1(y) + C\left[(1+\lambda)\int_0^y t(y-t)^{1/2} E_{3/2,3/2}\left[(1+\lambda)(y-t)^{3/2}\right]dt + y\right]$$

$$f_1(y) = (1+\lambda)\int_0^y \left[(y-t)^{1/2} E_{3/2,3/2}\left[(1+\lambda)(y-t)^{3/2}\right] \times \left[\int_0^t\int_0^z D_{0x}^{1/2} F(x)\,dxdz + \varphi_1(0)\right]\right]dt +$$

$$+\int_0^y\int_0^z D_{0t}^{1/2} F(t)\,dtdz + \varphi_1(0).$$

(22) according to the formula $\tau(h)$ va we calculate the $\int_0^h \tau(y)\,dy$:

$$\tau(h) = f_1(h) + C\left[(1+\lambda)\int_0^h t(h-t)^{1/2} E_{3/2,3/2}\left[(1+\lambda)(h-t)^{3/2}\right]dt + h\right] +$$

$$+\int_0^h R(h,s;1+\lambda,\mu)\left[f_1(s) + C(1+\lambda)\int_0^s t(s-t)^{1/2} E_{3/2,3/2}\left[(1+\lambda)(s-t)^{3/2}\right]dt + s\right]ds$$

,

$$\int_0^h \tau(y)\,dy = \int_0^h \left[f_1(y) + C\left[(1+\lambda)\int_0^y t(y-t)^{1/2} E_{3/2,3/2}\left[(1+\lambda)(y-t)^{3/2}\right]dt + y\right]\right]dy +$$

$$+\int_0^h R(y,s,1+\lambda,\mu)\left[f_1(s) + C\left[(1+\lambda)\int_0^s t(s-t)^{1/2} E_{3/2,3/2}\left[(1+\lambda)(s-t)^{3/2}\right]dt + s\right]\right]dsdy.$$

Putting these in the second condition (12), we arrive at the following equality:

$$C\left[(1+\lambda)\int_0^h t(h-t)^{1/2} E_{3/2,3/2}\left[(1+\lambda)(h-t)^{3/2}\right]dt + h + \right.$$

$$+\int_0^h R(h,s;1+\lambda,\mu)\cdot\left[(1+\lambda)\int_0^s t(s-t)^{1/2} E_{3/2,3/2}\left[(1+\lambda)(s-t)^{3/2}\right]dt + s\right]ds -$$

$$-a(0)\int_0^h\left[(1+\lambda)\int_0^y t(y-t)^{1/2} E_{3/2,3/2}\left[(1+\lambda)(y-t)^{3/2}\right]dt + y\right]dy -$$

$$\left.-a(0)\int_0^h\int_0^y R(y,s;1+\lambda,\mu)\left[(1+\lambda)\int_0^s t(s-t)^{1/2} E_{3/3,3/2}\left[(1+\lambda)(s-t)^{3/2}\right]dt + s\right]dsdy\right] =$$

$$= a(0)\int_0^h f_1(y)\,dy + a(0)\int_0^h\int_0^y R(y,s;1+\lambda,\mu) f_1(s)\,dsdy +$$

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$$+\varphi_3(0)-f_1(h)-\int_0^h R(h,s;1+\lambda,\mu)\,f_1(s)\,ds.$$

If $a(0)$ and h the following inequality for numbers

$$(1+\lambda)\int_0^h t(h-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(h-t)^{3/2}\right]dt+h+$$

$$+\int_0^h R(h,s;1+\lambda,\mu)\cdot\left[(1+\lambda)\int_0^s t(s-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(s-t)^{3/2}\right]dt+s\right]ds-$$

$$-a(0)\int_0^h\left[(1+\lambda)\int_0^y t(y-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(y-t)^{3/2}\right]dt+y\right]dy-$$

(23)

$$-a(0)\int_0^h\int_0^y R(y,s;1+\lambda,\mu)\left[(1+\lambda)\int_0^s t(s-t)^{1/2}E_{3/2,3/2}\left[(1+\lambda)(s-t)^{3/2}\right]dt+s\right]dsdy\neq 0$$

if done, (22) from equality, the unknown number is found to be one-valued.

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