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APPLICATION OF VECTORS IN SOLVING PLANIMETRIC PROBLEMS

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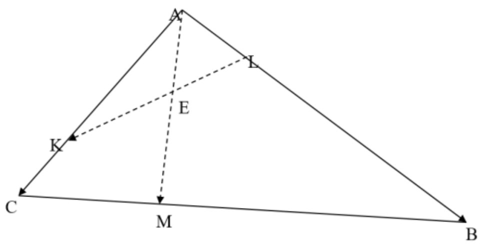
Abstract: This article analyzes the advantages and practical significance of using the vector method in solving planimetric problems. The vector method allows you to express the relationships between geometric figures in algebraic form, which simplifies and generalizes the process of solving problems. The article considers solutions to planimetric problems involving triangles, parallelograms, circles, and other geometric figures using vector operations - addition, subtraction, scalar and vector products. The vector method provides greater logical consistency compared to traditional geometric approaches, reduces calculation errors, and serves to develop students' spatial and analytical thinking. The results of the study show that the use of the vector method in the planimetry course is an effective tool for students to develop a deeper understanding of mathematical concepts and independent thinking skills.
Keywords: Vector, geometric problem, collinear vectors, noncollinear vectors

In the process of solving standard tasks, which are often found in problem sets and textbooks, expressed in the form of standard requirements (find, prove, calculate, replace), the competence under study is poorly formed. The problematic issue is based precisely on the lack of information, and its solution requires full work with this information. Students find themselves in conditions where they need to search for missing information, select the necessary one, analyze, organize, store, and sometimes communicate it. Geometric in matters problem solving find methods to bring known algorithms through done is increased . Currently modern in mathematics general middle education in schools mathematics system practical and practical in character issues system important importance profession will . In this students first of all learned mathematician concepts system imagination as then him/her in practice application priority from tasks one counted coming . Below geometry science planimetry (geometry on the plain forms properties learner section) to the section related issues in solution vectors algebra from the elements used .

Problem 1. *Point K is located* on the side of a triangle BCsuch that BM = 2 CM , points K and L are chosen from sides AC and AB respectively, such that AK ABC= *2* CK , BL = 3 AL , KL divides *the* section AM into what proportions?

Solution : $\overrightarrow{AB}$and we can define $\overrightarrow{AC}$the vectors briefly by $\vec{a}$and $\vec{b}$. $\overrightarrow{AE}$= x$\overrightarrow{AM}$, $\overrightarrow{LE}$= y$\overrightarrow{LK}$ Let it be .



1-chizma.

$$\overrightarrow{AM} = \vec{a} + \frac{2}{3}\,\overrightarrow{BC} = \vec{a} + \frac{2}{3}\left(\vec{b} - \vec{a}\right) = \frac{1}{3}\,\vec{a} + \frac{2}{3}\vec{b}$$

$$\overrightarrow{AE} = \frac{x}{3}\vec{a} + \frac{2x}{3}\vec{b}$$

other from the side

$\overrightarrow{AE} = \overrightarrow{AL} + y\,\overrightarrow{LK} = \frac{1}{4}\vec{a} + y(\frac{2}{3}\vec{b} - \frac{1}{4}\vec{a}) = \frac{1}{4}(1-y)\vec{a} + \frac{2y}{3}\vec{b}$

2 noncollinear vectors according to vector spread only in appearance since it was following equations system harvest will be :

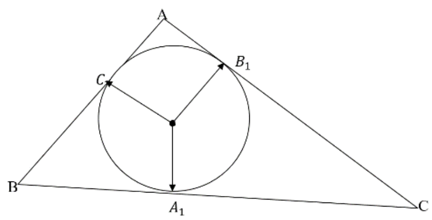
$$\frac{x}{3} = \frac{1}{4}(1-y)$$
$$\frac{2x}{3} = \frac{2y}{3}$$

here x =3/7. The desired ratio AE:is EM=3:4.

Answer : AE:EM=3:4

Issue 2. Optional In triangle ABC internal corners A, B, C if , $\cos A + \cos B + \cos C \leq \frac{3}{2}$ attitude appropriate that proof do it .

Solution : O point ABC to the triangle internal drawn circle center Let (drawing 2).



2-chizma.

$\overrightarrow{OA_1} + \overrightarrow{OB_1} + \overrightarrow{OC_1}| \geq c$

Hypothesis let's do this $|\overrightarrow{OA_1}| = r$

$$|\overrightarrow{OA_1}| = |\overrightarrow{OB_1}| = |\overrightarrow{OC_1}| = r$$
$$|\overrightarrow{OA_1} + \overrightarrow{OB_1} + \overrightarrow{OC_1}|^2 \geq 0$$

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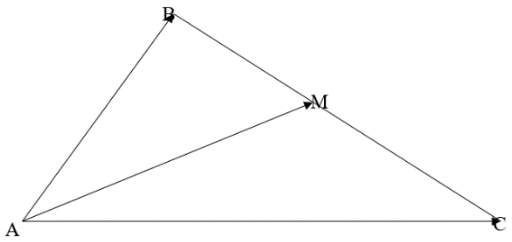
$$\overrightarrow{OA_1}^2 + \overrightarrow{OB_1}^2 + \overrightarrow{OC_1}^2 + 2\cdot|\overrightarrow{OA_1}|\cdot|\overrightarrow{OB_1}|\cdot\cos(180-C) + 2\cdot|\overrightarrow{OA_1}|\cdot|\overrightarrow{OC_1}|\cdot$$
$$\cdot\cos(180-B) + 2\cdot|\overrightarrow{OB_1}|\cdot|\overrightarrow{OC_1}|\cdot\cos(180-A) \geq 0$$

$3r^2 - 2r^2(\cos C + \cos B + \cos A) \geq 0$

$$\cos C + \cos B + \cos A \leq \frac{3}{2}$$

Issue 3. *ABC* triangle *A, B, C* corners given is , *M* point *BC* side middle if , *BAM* the corner count .

Solution : Hypothesis Let's do it , let's do it $< BAM = \varphi$(drawing 3) $AB = c, AC = b, BC = a$.



3-chizma.

$\overrightarrow{AM} \uparrow\uparrow (\overrightarrow{AB} + \overrightarrow{AC})$because

$$\cos\varphi = \frac{(\overrightarrow{AB}\cdot(\overrightarrow{AB}+\overrightarrow{AC}))}{|\overrightarrow{AB}|\cdot|\overrightarrow{AB}+\overrightarrow{AC}|} = \frac{\overrightarrow{AB}^2 + (\overrightarrow{AB},\overrightarrow{AC})}{c\cdot\sqrt{c^2+b^2+2\cdot b\cdot c\cos A}} == \frac{c + b\cdot\cos A}{\sqrt{c^2+b^2+2\cdot b\cdot c\cos A}}$$

and $\frac{b}{c} = \frac{\sin B}{\sin C}$from

$$\cos\varphi = \frac{\sin C + \sin B\cdot\sin A}{\sqrt{\sin^2 B + \sin^2 C + \sin^2 A}}.$$

So,

$$\varphi = arc\cos\frac{\sin C + \sin B\cdot\sin A}{\sqrt{\sin^2 B + \sin^2 C + \sin^2 A}}$$

Answer : $= arc\cos\frac{\sin C+\sin B\cdot\sin A}{\sqrt{\sin^2 B+\sin^2 C+\sin^2 A}}$.

Problem 4: Parallelogram diagonals squares sum his/her all sides squares to the sum equality prove it .

Solution : $\overrightarrow{AB} = \overrightarrow{DC} = \vec{a}$, $\overrightarrow{AD} = \overrightarrow{BC} = \vec{b}$, $\overrightarrow{AC} = \overrightarrow{d_1}$, $\overrightarrow{BD} = \overrightarrow{d_2}$,$< BAD = \varphi$ Let (drawing 4).

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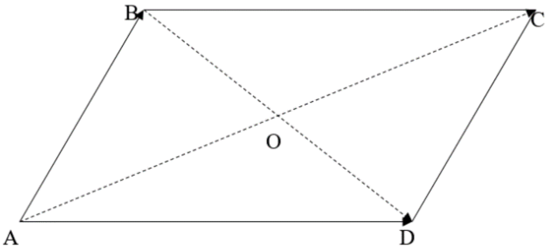


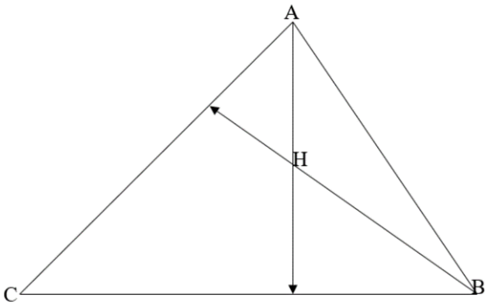
4-chizma.

$$\begin{cases}\vec{a}+\vec{b}=\overrightarrow{d_1}\\ \vec{a}-\vec{b}=\overrightarrow{d_2}\end{cases}\Rightarrow\begin{cases}\vec{a}^2+2\cdot|\vec{a}|\cdot|\vec{b}|\cdot\cos\varphi+\vec{b}^2=\left|\overrightarrow{d_1}\right|^2\\ \vec{a}^2-2\cdot|\vec{a}|\cdot|\vec{b}|\cdot\cos\varphi+\vec{b}^2=\left|\overrightarrow{d_2}\right|^2\end{cases}\Rightarrow 2\cdot\vec{a}^2+2\cdot\vec{b}^2==\overrightarrow{d_1}^2+\overrightarrow{d_2}^2$$

$$\Rightarrow\overrightarrow{d_1}^2+\overrightarrow{d_2}^2=2\cdot\left(\vec{a}^2+\vec{b}^2\right).$$

Problem 5: Triangle heights one on point intersection prove .

Solution : H - triangle A and B from the ends held two height intersection point (Figure 5). Therefore , point H third to the height relevant that proof need , that is $CH \perp AB$.



5-chizma.

$\overrightarrow{AH} \perp \overrightarrow{BC}$ and $\overrightarrow{BH} \perp \overrightarrow{CA}$ since it is

$\overrightarrow{AH}\cdot\overrightarrow{BC}=0,\ \overrightarrow{BH}\cdot\overrightarrow{CA}=0.$

$\overrightarrow{BC}=\overrightarrow{BH}+\overrightarrow{HC}=\overrightarrow{BH}-\overrightarrow{CH},\ \overrightarrow{CA}=\overrightarrow{CH}-\overrightarrow{AH}.$

$\overrightarrow{AH}\cdot\left(\overrightarrow{BH}-\overrightarrow{CH}\right)=0,\overrightarrow{BH}\cdot\left(\overrightarrow{CH}-\overrightarrow{AH}\right)=0$

Equality adding

$$\overrightarrow{CH}\cdot\left(\overrightarrow{BH}-\overrightarrow{AH}\right)=0$$

what harvest we will do it , from now on

$$\overrightarrow{CH}\cdot\overrightarrow{BA}=0.$$

So, all three altitudes intersect at one point.

Problem 6: Triangle sides $a,$ b and c to equal to . a towards held median $m_a=\frac{1}{2}\sqrt{2\cdot c^2+2\cdot b^2-a^2}$ Prove that it is calculated by the formula.

Solution : $BC=a, CA=b, AB=c, AM=m_a$. M point ABC triangle BC side of middle Let it be . AM the line any D to the point continue , forming a parallelogram $ABCD$ (Figure 6) .

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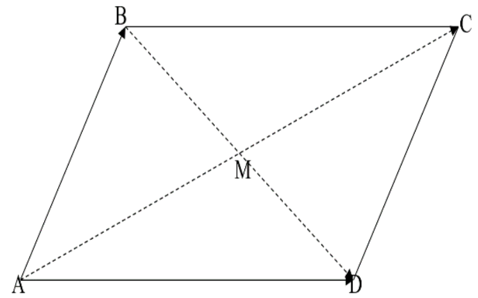

6-chizma.

Vectors add and subtraction to the rules based the following harvest we do :

$\overrightarrow{AB} + \overrightarrow{AC} = 2 \cdot \overrightarrow{AM}$ $\quad |\overrightarrow{AB}|^2 - 2 \cdot |\overrightarrow{AB}| \cdot |\overrightarrow{AC}| \cdot \cos\alpha + |\overrightarrow{AC}|^2 = 4 \cdot |\overrightarrow{AM}|^2$

$\overrightarrow{AC} - \overrightarrow{AB} = \overrightarrow{BC}$

$$|\overrightarrow{AC}|^2 - 2 \cdot |\overrightarrow{AC}| \cdot |\overrightarrow{AB}| \cdot \cos\alpha + |\overrightarrow{AB}|^2 = |\overrightarrow{BC}|^2$$

$$|\vec{c}|^2 - 2 \cdot |\vec{b}| \cdot |\vec{c}| \cdot \cos\alpha + |\vec{b}|^2 = 4 \cdot |\overrightarrow{m_a}|^2$$
$$|\vec{b}|^2 + 2 \cdot |\vec{b}| \cdot |\vec{c}| \cdot \cos\alpha + |\vec{c}|^2 = |\vec{a}|^2$$
$$2 \cdot |\vec{b}| \cdot |\vec{c}| \cdot \cos\alpha = |\vec{a}|^2 - |\vec{b}|^2 - |\vec{c}|^2$$

Harvest was from equality using to the following has we will be :

$$|\vec{c}|^2 - |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + |\vec{b}|^2 = 4 \cdot |\overrightarrow{m_a}|^2 \Rightarrow$$

$$\Rightarrow 2 \cdot |\vec{b}|^2 + 2 \cdot |\vec{c}|^2 - |\vec{a}|^2 = 4 \cdot |\overrightarrow{m_a}|^2$$

from this,

$$m_a = \frac{1}{2}\sqrt{2 \cdot c^2 + 2 \cdot b^2 - a^2}$$

come comes out

Problem 7: *ABCD* convex of a rectangle *AC* and *BD* diagonals *F* on point If $|AF| = |CF| = 2, |BF| = 1, |DF| = 4, \angle BFC = \frac{\pi}{3}$, if *AB* and *DC* sides between Find the angle .

Solution : Let $|AF| = |CF| = 2, |BF| = 1, |DF| = 4, \angle BFC = \frac{\pi}{3}$, (AB^DC)= φ(Figure 7).

$$\overrightarrow{AB} = \overrightarrow{AF} + \overrightarrow{FB}$$

$$|\overrightarrow{AB}| = \sqrt{(\overrightarrow{AF} + \overrightarrow{FB})^2} = \sqrt{\overrightarrow{AF}^2 + 2 \cdot |\overrightarrow{AF}| \cdot |\overrightarrow{FB}| \cdot \cos 60° + \overrightarrow{FB}^2} == \sqrt{4 + 2 \cdot 2 \cdot 1 \cdot \frac{1}{2} + 1}$$
$$= \sqrt{7}$$

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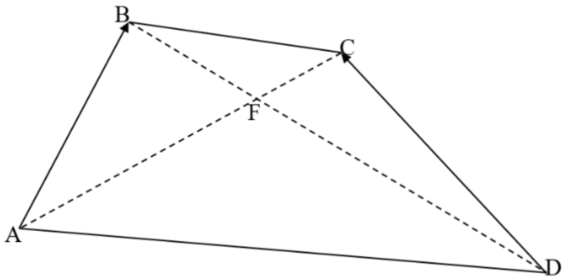

7-chizma.

$$\overrightarrow{DC} = \overrightarrow{DF} + \overrightarrow{FC}$$

$$|\overrightarrow{DC}| = \sqrt{(\overrightarrow{DF} + \overrightarrow{FC})^2} = \sqrt{\overrightarrow{DF}^2 + 2 \cdot |\overrightarrow{DF}| \cdot |\overrightarrow{FC}| \cdot \cos 60° + \overrightarrow{FC}^2} =$$

$$= \sqrt{16 + 2 \cdot 4 \cdot 2 \cdot \frac{1}{2} + 4} = 2\sqrt{7}$$

$$\overrightarrow{AB} \cdot \overrightarrow{DC} = (\overrightarrow{AF} + \overrightarrow{FB}) \cdot (\overrightarrow{DF} + \overrightarrow{FC}) = 4 + 4 + 4 + 1 = 13$$

$$\cos\varphi = \frac{\overrightarrow{AB} \cdot \overrightarrow{DC}}{|\overrightarrow{AB}| \cdot |\overrightarrow{DC}|} = \frac{13}{\sqrt{7} \cdot 2\sqrt{7}} = \frac{13}{14}$$

$$\varphi = arc\cos\frac{13}{14}.$$

Answer : $\varphi = arc\cos\frac{13}{14}$.

Independent solution for issues .

Problem 8: Let $M$and Nbe the midpoints of the sides $AMND$of the rectangle $AB$and , CDrespectively . $ABCD$Prove that the midpoints of the diagonals of the rectangle and are the vertices of a parallelogram and that they lie on the same line .$BMNC$

Problem 9: The points $M$and are $N$the vertices of the rectangle $AD$and , BCrespectively .$ABCD$ of the parties middle points Let . $AB$and CDas a result of continuing the sides they Pintersect at a point. Prove that the points lie on one line $ABCD$only if the rectangle is a trapezoid M, N va P.

Issue 10:l_1 and The lines $l_2$are intersected by three parallel lines at points $\frac{AC}{CB} = \frac{A_1C_1}{C_1B_1}$and A_1, B_1, C_1respectively . A, B, CProve that the ratio holds.

Problem 11: Triangle medians one on point intersection and intersection at the point triangle from the end when counting $2:1$Prove that it is divisible by .

Issue 12: D point ABC equal sideways triangle AB of the base middle point , DM line BC towards perpendicular ($M\epsilon BC$), N $-DM$ of the line Prove that the midpoints of the lines are perpendicular .AM va CN

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Problem 13: ABCIn a triangle, its altitude CC_1 $-$is a point $AB.C_1$ side how in proportion Find the separation .

Vectors method planimetry issues in solution effective , logical consistent and generalized approach This is method using geometric of figures main properties algebraic in the form is expressed , this and issues analysis to do and to prove much simplifies . Vector approach not only of the students mathematician level increases , maybe their spatial thinking , analysis to do and independent work skills to develop help gives . Therefore planimetry in their classes vectors method application education efficiency increasing important from factors one is considered .

Foydalanilgan adabiyotlar ro'yxati

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