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APPLICATIONS OF LINE INTEGRALS TO AMPERE'S AND FARADAY'S LAWS

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Abstract: In this article, the scientific significance of line integrals of the second kind, which are one of the fundamental concepts of mathematical analysis, in expressing the laws of electrodynamics is investigated. It is demonstrated that line integrals serve as the primary computational tool in studying processes occurring in vector fields. The main focus is on the mathematical model of Ampere's and Faraday's laws and their application in solving specific physical problems, particularly the problem of calculating the magnetic field of a current-carrying straight conductor. It is thoroughly analyzed how the fundamental laws of electrodynamics lead to classic formulas through a purely logical-mathematical approach.
Keywords: Line integral, Ampere's law, Faraday's law, circulation, vector field, electromagnetic induction, mathematical modeling.

A high-level mathematical apparatus is required for accurate calculation of processes occurring in modern engineering and technical sciences. In classical mechanics, while the work done by a constant force along a straight path is primarily calculated using the formula $A = F \cdot S \cdot \cos\alpha$. , this approach loses its effectiveness when the trajectory of motion and the field are variable. Most complex physical quantities in nature and technology (impulse of force, mechanical work, electromotive force, magnetic flux) are inherently linked to vector fields. In studying the properties of these fields, a special tool of mathematical analysis – line integrals – plays a decisive role. The main goal of this research is to mathematically model Ampere's and Faraday's laws, which are the foundation of electromagnetism, using line integrals of the second kind, and to demonstrate their analytical solutions in practice.

If, in curvilinear motion, direction along with magnitude is also significant, a line integral of the second type is used for the solution of the problem. This integral represents the work done by the field along the curve or its circulation.

Suppose that a material point is moving along the curve L in a force field $\vec{F}(x,y) = P(x,y)\vec{i} + Q(x,y)\vec{j}$. The mechanical work done in this process is expressed by the following definite integral:

$$A = \int_L \vec{F} \cdot d\vec{r} = \int_L P(x,y)dx + Q(x,y)dy$$

In electromagnetism, this concept extends as the circulation of a vector field. Circulation characterizes the "rotational" movement of the field along a closed contour.

Ampere's law, which is the basis of Maxwell's equations, can be directly derived through a line integral. According to Ampere's law: The circulation of the magnetic induction vector $\vec{B}$ taken along a closed contour L is proportional to the algebraic sum of the currents enclosed by that contour[2, 6].

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Its rigorous mathematical expression is written as follows:

$$\oint_L \vec{B}\cdot d\vec{l} = \mu_0 \sum I_k$$

Here, μ_0 is the magnetic constant. This fundamental formula is a powerful basis for calculating the magnetic field of conductors of various shapes in engineering practice. We can see this in the analysis of the following classic example:

Example (Calculating the magnetic field of a current-carrying straight conductor): A current I flows through an infinitely long straight conductor. It is required to find the magnetic induction at a point at a distance R from the conductor.

1. According to the symmetry of the problem, magnetic field lines form concentric circles around the conductor and the vector $\vec{B}$ is directed tangentially (see Fig. 1).
2. For calculation, we take a circle of radius R as the closed contour L.
3. Since the directions of $\vec{B}$ and $d\vec{l}$ coincide (are parallel) under the integral, the scalar product becomes the product of their magnitudes: $\vec{B}\cdot d\vec{l} = B\cdot dl$. Also, the magnitude B is constant along this circle.
4. We take the constant magnitude out of the integral sign and integrate:

$$\oint_L Bdl = B\oint_L dl = B\cdot 2\pi R$$

5. We substitute the obtained result into Ampere's Law:

$$B\cdot 2\pi R = \mu_0 I$$

From this, the famous calculation formula used by physicists worldwide is derived:

$$B = \frac{\mu_0 I}{2\pi R}$$

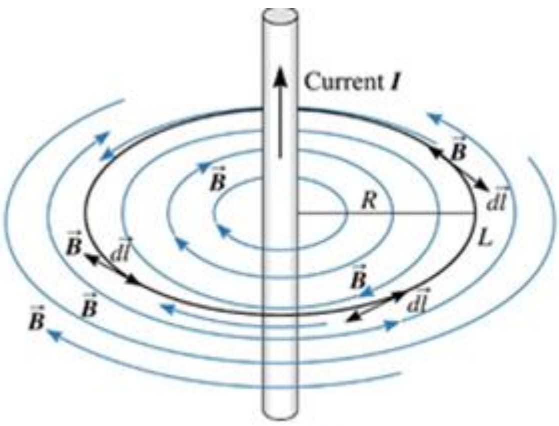

Figure 1

This solution shows how convenient and rigorous it is to derive fundamental physical laws using line integrals.

Another invaluable application of the line integral is related to calculating the electromotive force (EMF) in dynamic conditions. According to Faraday's law of electromagnetic induction, a changing magnetic field generates a swirling electric field in space. Expressing the induced EMF (ò) in a closed contour precisely as the circulation of the electric field strength $\vec{E}$ is the only scientifically correct way. Its analytical expression is written through integrals as follows:

$$ò = \oint_L \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt}$$

Here, Φ is the magnetic flux passing through the given contour, and its derivative with respect to time $\frac{d\Phi}{dt}$ gives the rate of change of the field. To clearly understand this principle, let's consider the following mathematical modeling problem:

Example (Calculation of an Eddy Electric Field Generated in a Varying Magnetic Field):

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A closed loop conductor with radius R is located in a spatial plane. The induction of a uniform magnetic field, perpendicular to the surface of this loop, increases over time according to the linear law $B(t) = k \cdot t$ (where k is a constant representing the rate of change of the field). Calculate the induced electromotive force (ò) and the magnitude of the electric field strength (E) along the conductor's contour [1, 3].

1. First, we find the total magnetic flux Φ passing through the loop's surface (S) using a double integral. Since the field is uniform and perpendicular to the surface, the integration simplifies considerably:

$$\Phi = \iint_S \vec{B} \cdot d\vec{S} = B(t) \cdot S = (k \cdot t) \cdot (\pi R^2)$$

2. To find the right side of Faraday's law, we take the derivative of this flux with respect to time

$$(t\ ): -\frac{d\Phi}{dt} = -\frac{d}{dt}(k \cdot \pi R^2 \cdot t) = -k\pi R^2$$

This value is equal to the induced EMF generated in the loop: ò $= -k\pi R^2$. The negative sign, according to Lenz's law, indicates that the generated field opposes the external change.

3. Now we calculate the left side of the law the circulation of the electric field. As illustrated in Figure 2, according to symmetry principles, a varying magnetic field generates electric field lines in the form of concentric circles around itself. Therefore, the vector $\vec{E}$ will be directed along the tangent to the loop, and its magnitude will be the same at all points of the contour ($E = \text{const}$). Opening the scalar product, we calculate the integral:

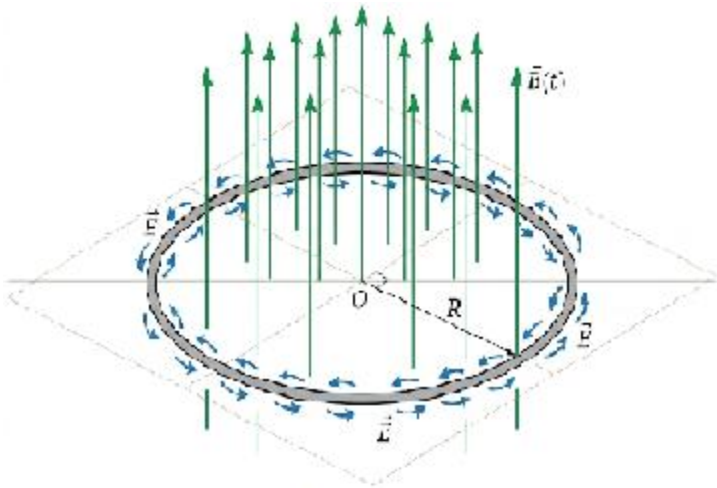

Figure 2

$$\oint_L \vec{E} \cdot d\vec{l} = \oint_L E \cdot dl = E \oint_L dl = E \cdot (2\pi R)$$

4. We equate both sides of Faraday's law equation:

$$E \cdot 2\pi R = |-k\pi R^2|$$

5. As a result, we obtain the exact formula for the electric field strength:

$$E = \frac{kR}{2}$$

This mathematical analysis and example prove that the time derivative of a varying magnetic field is directly related to the curvilinear integral (circulation) over space. Using the same apparatus, electrical processes in not only simple loops but also all types of complex coils (transformers and generators) in engineering practice are modeled.

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Mathematical and physical analyses conducted prove that line integrals of the second kind are not merely dry mathematical abstractions, but rather one of the most powerful tools for studying the geometric and physical properties of the real world (work, electromagnetic field circulation, energy exchange). The study rigorously established that the line integral is the primary tool for working with vector fields (force, electric, and magnetic fields). The fact that the work done in a variable force field, the circulation of the magnetic field according to Ampere's law, and Faraday's laws of electromagnetic induction are directly expressed in terms of line integrals clearly confirms how intimately the science of mathematics is connected with physics and engineering sciences. In the future, applying this apparatus to model the diffraction of electromagnetic waves within composite materials of complex spatial shapes will yield high practical effectiveness.

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