

APPLICATION OF THE PROBABILITY ADDITION THEOREM IN PRACTICAL PROBLEMS

Rajabov U.T.

National Pedagogical University of Uzbekistan named after Nizami,
Uzbekistan; ulugbekrajabov0330@gmail.com

Davletbayev SH.D.

National Pedagogical University of Uzbekistan named after Nizami, Uzbekistan;
davletbayevsherdod@gmail.com

Abstract: This article explores the theoretical foundations of the probability addition theorem and its application in practical problems. The article presents probability addition formulas for mutually exclusive and non-mutually exclusive events, and their application in various fields such as transport and logistics, medicine and biology, economics and financial risk assessment, and education and pedagogy is explained through examples. Practical problems for each field are analyzed along with their solutions, revealing the importance of probability theory in real life. The research results show that the probability addition theorem is an important mathematical tool in evaluating complex processes, identifying risks, and making decisions.

Keywords: Probability theory, Probability addition theorem, Mutually exclusive events, Non-mutually exclusive events, Practical problems, Transport logistics, Medical statistics, Financial risks, Pedagogical analysis, Probability calculations, Random events.

We will consider the probability addition theorem in the following cases:

Theorem: The probability of either of two mutually exclusive events occurring is equal to the sum of their individual probabilities.

$$P(A + B) = P(A) + P(B).$$

Theorem: The probability of at least one of two non-mutually exclusive events occurring is equal to the sum of their probabilities minus the probability of their simultaneous occurrence.

$$P(A + B) = P(A) + P(B) - P(A \cdot B)$$

For mutually exclusive events, i.e., if they do not occur simultaneously, the addition theorem has the following form:

$$P(A + B) = P(A) + P(B)$$

And for n mutually exclusive events:

$$P(A_1 + A_2 + \dots + A_n) = P(A_1) + P(A_2) + \dots + P(A_n)$$

For events that are not mutually exclusive, i.e., they can occur simultaneously. In this case:

$$P(A + B) = P(A) + P(B) - P(A \cdot B)$$

Here $P(A \cdot B)$ is the probability of both events occurring simultaneously. For three events:

$$P(A + B + C) = P(A) + P(B) + P(C) - P(A \cdot B) - P(A \cdot C) - P(B \cdot C) + P(A \cdot B \cdot C)$$

By using these formulas, many real-world problems can be solved. Below, practical problems from various fields and their solutions are presented.

1. Application in transport and logistics

In the transport system, the addition theorem of probabilities is widely used. Specifically, this theorem is used to calculate the probabilities of road traffic accidents, flight delays, and losses in cargo transportation.

For mutually exclusive events:

Problem 1. A city bus travels along its route. The causes of delay can be only one: traffic jam (A), technical malfunction (B), or rain (C). These causes cannot occur simultaneously (mutually exclusive).

$$P(A) = 0,30; \quad P(B) = 0,12; \quad P(C) = 0,08$$

Find the probability that the bus will be delayed by at least one cause.

Solution: The events are mutually exclusive, therefore:

$$P(A+B+C) = P(A) + P(B) + P(C)$$

$$P(A+B+C) = 0,30 + 0,12 + 0,08 = 0,50$$

Answer: The probability of the bus being delayed is 0.5.

For dependent events (events that can occur together):

$$P(A+B) = P(A) + P(B) - P(A \cdot B)$$

Problem 1. A city bus travels along its route. The probability of a passenger boarding at the first stop is 0.7, and the probability of a passenger boarding at the second stop is 0.4. The probability of both events occurring simultaneously (the bus stopping at both stops) is 0.3. Find the probability that the bus will stop at at least one stop for passengers to board.

Solution: Let A be the event that a passenger boards at the first stop, and B be the event that a passenger boards at the second stop. Here

$$P(A) = 0,7; \quad P(B) = 0,4; \quad P(A \cdot B) = 0,3$$

$$P(A+B) = P(A) + P(B) - P(A \cdot B) = 0,7 + 0,4 - 0,3 = 0,8$$

Answer: The probability that the bus stops at at least one stop for a passenger to board is 0.8.

Problem 2. Three problems can occur simultaneously at a cargo port: crane malfunction (A), ship delay (B), document issue (C).

$$P(A) = 0,18; \quad P(B) = 0,22; \quad P(C) = 0,14$$

$$P(A \cdot B) = 0,07; \quad P(A \cdot C) = 0,05; \quad P(B \cdot C) = 0,06; \quad P(A \cdot B \cdot C) = 0,02$$

Find the probability that at least one problem occurs at the port.

Solution:

$$P(A+B+C) = P(A) + P(B) + P(C) - P(A \cdot B) - P(A \cdot C) - P(B \cdot C) + P(A \cdot B \cdot C) = 0,18 + 0,22 + 0,14 - 0,07 - 0,05 - 0,06 + 0,02 = 0,38$$

Answer: The probability that at least one problem occurs at the port is 0.38 (38%).

2. Application in medicine and biology

In medical practice, probability theory plays an important role in diagnosing diseases, evaluating the effectiveness of treatment methods, and in epidemiological research. The theorem of addition of probabilities is especially applied in calculating the probability of several symptoms or diseases occurring together.

For mutually exclusive events

Problem 1. A patient admitted to the hospital can only be diagnosed with one condition: influenza (A), bronchitis (B), or allergy (C). These causes do not occur simultaneously.

$$P(A) = 0,35; \quad P(B) = 0,25; \quad P(C) = 0,15$$

Find the probability that the patient is diagnosed with one of these diseases.

$$\text{Solution: } P(A + B + C) = 0,35 + 0,25 + 0,15 = 0,75$$

Answer: The probability of being diagnosed with one of these three diseases is 0.75.

For dependent events

Problem 1. In laboratory analysis, the probability of an error occurring during blood group determination is 0.02, and the probability of an error in tissue compatibility determination is 0.03. The probability of both types of errors occurring simultaneously is very small (0.001). Find the probability that at least one error occurs.

Solution:

$$P(A + B) = 0,02 + 0,03 - 0,001 = 0,049$$

Answer: The probability of at least one error occurring in laboratory analysis is approximately 0.049. This should be taken into account in medical quality control.

Problem 2. After surgery, a patient may experience complication A (probability 0.20), complication B (probability 0.15), or complication C (probability 0.10).

$$P(A \cdot B) = 0,06; \quad P(A \cdot C) = 0,04; \quad P(B \cdot C) = 0,03; \quad P(A \cdot B \cdot C) = 0,01$$

Find the probability that the patient experiences at least one complication.

Solution:

$$P(A + B + C) = 0,20 + 0,15 + 0,10 - 0,06 - 0,04 - 0,03 + 0,01 = 0,33$$

Answer: The probability of at least one complication occurring is 0.33.

3. Application in economics and financial risk assessment

In modern economics, probability theory is an important tool in assessing financial risks and making investment decisions. Banks and insurance companies use the addition theorem of probabilities to calculate the probabilities of incurring loss or compensating for damage.

For independent events:

Problem 1. An insurance company receives three types of events during the year, only one of which can occur: fire (A), flood (B), theft (C).

$$P(A) = 0,005; \quad P(B) = 0,008; \quad P(C) = 0,012$$

Find the probability of an insured event occurring during the year. Yechim:

$$P(A + B + C) = 0,005 + 0,008 + 0,012 = 0,025$$

Answer: The probability of an insured event occurring during the year is 0.025 (2.5%).

Dependent event (joint events)

Problem 1. According to the insurance company's data, the probability of a mudslide in the region during the year is 0.12, and the probability of a storm is 0.18. The probability of both events occurring in the same year is 0.04. Find the probability that at least one of these natural disasters occurs.

Solution: A – mudslide, B – storm events.

$$P(A) = 0,12; \quad P(B) = 0,18; \quad P(A \cdot B) = 0,04$$

$$P(A + B) = 0,12 + 0,18 - 0,04 = 0,26$$

Answer: The probability of at least one natural disaster occurring is 0.26 (26%). The insurance company determines the insurance amount taking this probability into account.

4. Application in education and pedagogy

In the educational process, the addition theorem of probabilities is used to analyze students' test results, calculate the probability of answering a question correctly, and assess the level of understanding of the learning material.

For independent events:

Problem 1. In an Olympiad test, a student can take 1st, 2nd, or 3rd place (mutually exclusive events).

$$P_{(I \text{ orin})} = 0,10; \quad P_{(II \text{ orin})} = 0,20; \quad P_{(III \text{ orin})} = 0,25$$

Find the probability that the student will get a prize-winning place (I, II, or III).

Solution:

$$P = 0,10 + 0,20 + 0,25 = 0,55$$

Answer: The probability that the student will get a prize-winning place is 0.55.

Problem 2. In a test, there is only one correct answer for each question (A, B, C, D). The probability that a student answers the first question correctly is 0.80; for the second question, it is 0.70; and for the third question, it is 0.60. Each question is considered separately, and one question is solved at a time. Find the probability that the student answers at least one question correctly (using the complementary event).

Solution:

Probability of not answering any correctly:

$$P_{(None)} = (1 - 0,80) \cdot (1 - 0,70) \cdot (1 - 0,60) = 0,20 \cdot 0,30 \cdot 0,40 = 0,024$$

$$P_{(atleastone)} = 1 - 0,024 = 0,976$$

Answer: The probability that the student answers at least one question correctly is 0.976.

For dependent event:

Problem 1. There are 30 students in the class: 18 of them like mathematics (A), 15 of them like computer science (B), and 10 of them like both subjects. Find the probability that a randomly selected student likes mathematics or computer science.

Solution:

$$P(A) = \frac{18}{30} = 0,60; \quad P(B) = \frac{15}{30} = 0,50; \quad P(A \cdot B) = \frac{10}{30} \approx 0,33$$

$$P(A + B) = 0,60 + 0,50 - 0,33 = 0,77$$

Answer: The probability that a random student likes mathematics or computer science is 0.77.

The mathematical analyses carried out prove that the addition theorem of probabilities is an important mathematical tool in solving various practical problems. With the help of this theorem, the probabilities of events occurring in transport, medicine, economics, and education sectors are accurately calculated. The article demonstrated the application of formulas for mutually inclusive and mutually exclusive events through examples. Practical problems further

clearly demonstrated the importance of probability theory in real life. The obtained results confirm that probabilistic methods are effective in decision-making and risk assessment.

References

1. Gmurman V.E. Ehtimollar nazariyasi va matematik statistikadan masalalar yechishga doir qo'llanma. Toshkent: O'qituvchi, 1980. 365 b.
2. Abdushukurov A.A. Ehtimollar nazariyasi va matematik statistika: O'quv qo'llanma. Toshkent: Universitet, 2010. - ISBN: 978-9943-305-21-2.
3. Fayzullayeva S.F. Ehtimollar nazariyasidan masalalar to'plami: O'quv qo'llanma. - Toshkent: O'zbekiston faylasuflari milliy jamiyati nashriyoti, 2006. - 112 b.
4. Gnedenko B.V. Ehtimollar nazariyasi kursi. – Moskva: Nauka, 1988. – 448 b.
5. Sevastyanov B.A., Chistyakov V.P., Zubkov A.M. Ehtimollar nazariyasi. – Moskva: Nauka, 1985. – 304 b.
6. Feller W. Ehtimollar nazariyasiga kirish va uning tatbiqlari. I jild. – Moskva: Mir, 1984. – 528 b.
7. Kolmogorov A.N. Ehtimollar nazariyasining asoslari. – Moskva: Nauka, 1974. – 120 b.
8. Borovkov A.A. Ehtimollar nazariyasi. – Novosibirsk: Nauka, 1986. – 432 b.
9. Chistyakov I.I. Kombinatorika. – Moskva: Nauka, 1982. – 144 b.
10. Tulaganov A.T., Norqo'ziyev R.X. Ehtimollar nazariyasi va matematik statistika. – Toshkent: O'qituvchi, 2005. – 288 b.
11. Yusupov I.Q., Rashidov A.R. Ehtimollar nazariyasi va matematik statistikadan masalalar to'plami. – Toshkent: Fan, 2003. – 224 b.