Western European Journal of Historical Events and Social Science
Volume 2, Issue 5, May, 2024
https://westerneuropeanstudies.com/index.php/4
ISSN (E): 2942-1926 Open Access| Peer Reviewed

STUDYING THE MOVEMENT (PRESESSION) OF THE GYROSCOPIC AXIS AND SOLVING RELATED PROBLEMS

Mansur Dusmuratov
Associate Professor of Chirchik State Pedagogical University

Annotation: This article provides examples of solving problems related to the causes of precessional gyroscope motion, the causes of precessional angular velocity and the moment of precession force and related methods, as well as tasks that students can solve on their own.

Keywords: precession; precessional motion; precession period; angular velocity of precession; moment of precession; axis of rotation; rod; radius of gyration; center of mass; pivot; shaft.

As we all know, studying the laws of conservation of physics in physics and astronomy majors of pedagogical higher education institutions is of particular importance. Like other conservation laws, the study of the conservation of angular momentum and its various applications is very important not only in the field of mechanics, but also in atomic physics, nuclear physics, and quantum mechanics. Therefore, in this article, we will talk about one of the effects related to the law of conservation of angular momentum - the precessional movement of free gyroscopes and the precession effects that appear when the axis of rotation of the gyroscope is turned, as well as related techniques and life It is important to solve the problems encountered.

When we were little, most of us tried to open broken mechanical watches and play with the gears. At that time, when we held these wheels and turned them by their axis, we observed that at first the axis of the wheel was vertical, and a little later, the axis itself rotated as well. We observed a similar phenomenon when old women were spinning. Gear wheels of mechanical clocks or spinning objects like a ball are called gyroscopes in scientific language.

GIROSKOPIK O‘QINING HARAKATI (PRESESSIYA) NI O‘RGANISH VA UNGA OID MASALALAR YECHISH

Mansur Dusmuratov
Chirchiq davlat pedagogika universiteti dotsenti

Annotatsiya: *Ushbu maqolada giroskop presession harakatining yuzaga kelish sabablari, presession burchak tezlik hamda presession kuch momentini yuzaga kelish sabablari va ularga oid texnika bilan bog‘liq masalalar yechimidan namunalar hamda talabalar mustaqil yechish uchun masalalar berilgan.*

Калит сўзлар: *presessiya; presession harakat; presession davr; presession burchak tezlik; presession kuch momenti; aylanish o‘qi; sterjen; inersiya radiusi; massa markazi; tayanch; podshipnik; val.*

Western European Journal of Historical Events and Social Science
Volume 2, Issue 5, May, 2024
https://westerneuropeanstudies.com/index.php/4
ISSN (E): 2942-1926 Open Access| Peer Reviewed

Аннотация: *В этой статье приведены примеры решения проблем, связанных с причинами прецессионного движения гироскопа, причинами возникновения прецессионной угловой скорости и момента силы прецессии и связанными с ними методами, а также задачи, которые студенты могут решить самостоятельно.*

Ключевые слова: *прецессия; прецессионное движение; период прецессии; угловая скорость прецессии; прецессионный момент силы; ось вращения; стержень; радиус инерции; центр масс; опора; подшипник; вал.*

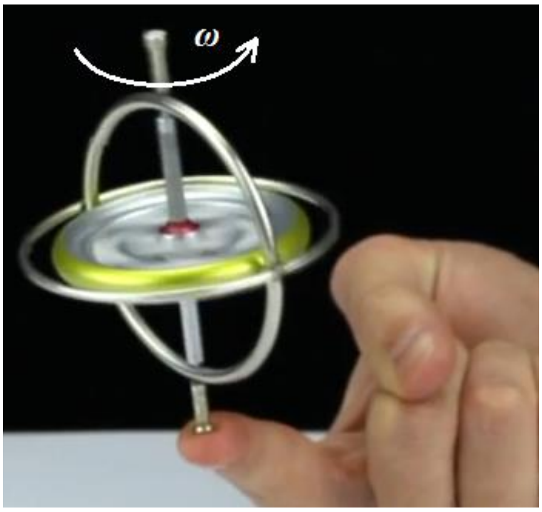


Figure 1. Invariance of the gyroscope axis

Figure 2. The law of conservation of angular momentum in a gyroscope

The term gyroscope was introduced to science in 1852 by Jean Foucault, and it was first coined by Johann Bonenberger in 1817. An elephant that children play with, a disk that rotates around its axis, a sphere, a cylinder, and similar objects are all examples of gyroscopes. The most remarkable feature of a gyroscope is that it does not change the direction of its axis when it rotates rapidly, that is, it keeps the direction stable. This can be explained by the law of conservation of angular momentum (Fig. 1).

If we take a free gyroscope in our hands and move it forward (up, down and side), the inner and outer frames are stationary, the axis of the rotor moves parallel to itself, but its axis does not turn. This can be explained by the law of conservation of angular momentum. Angular velocity vector when the disc rotates $\vec{\omega}$ and the angular momentum vector $\vec{L} = I\vec{\omega}$ is directed along the axis of the disc and during the rotational movement $\vec{L} = const$ tends to be (Fig. 2). Due to this same property, a flying elephant, a rocket, and other rotating objects tend to maintain the direction of the axis of rotation and tend to point in one direction in space. Free gyroscopes can be used as directional compasses in spacecraft, aviation, and marine applications.

Western European Journal of Historical Events and Social Science
Volume 2, Issue 5, May, 2024
https://westerneuropeanstudies.com/index.php/4
ISSN (E): 2942-1926 Open Access| Peer Reviewed

Now let's look at what happens when a spinning disk in a free gyroscope is forced to turn its axis. For this, we try to act on the frame of the free gyroscope with a constant force F. As a simple example, a load of weight F=mg is hung on one end of the gyroscope frame as shown in Fig. 3-a. In this case, in addition to the initial fast rotational movement ω around the core of the disk under the influence of the suspended load, a slow rotational movement Ω around the vertical axis is also added. To quantify this, we use Figure 3b. Placed outside $\vec{F}$ relative to the bottom support of the gyroscope under the influence of the force vector $\vec{M} = \vec{r} \times \vec{F}$ a torque vector equal to This moment of force is elementary during time dt

$$d\vec{L} = \vec{M}dt = \left(\vec{r} \times \vec{F}\right)dt \quad (2)$$

creates an increase in momentum equal to This causes the initial momentum vector to change, the disk axis

$$d\theta = \frac{dL}{L_0} = \frac{rFdt}{L_0} \quad (3)$$

causes it to turn a corner. As a result, the entire gyroscope revolves around the vertical Oz axis

$$\Omega = \frac{d\theta}{dt} = \frac{rF}{L_0} \quad (4)$$

the corner begins to turn rapidly.

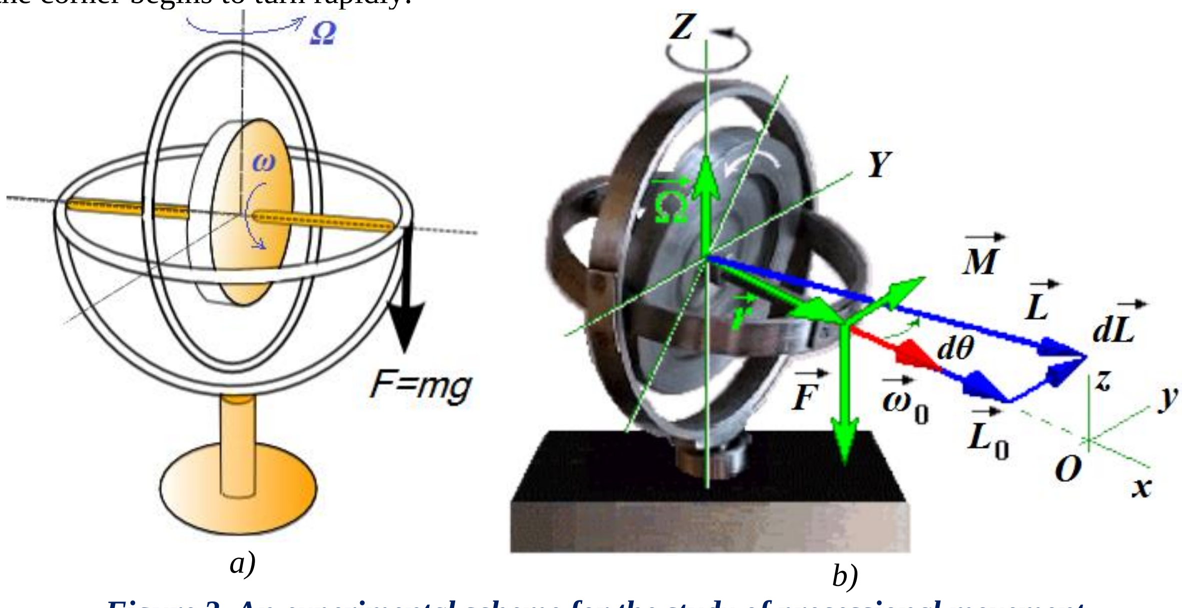


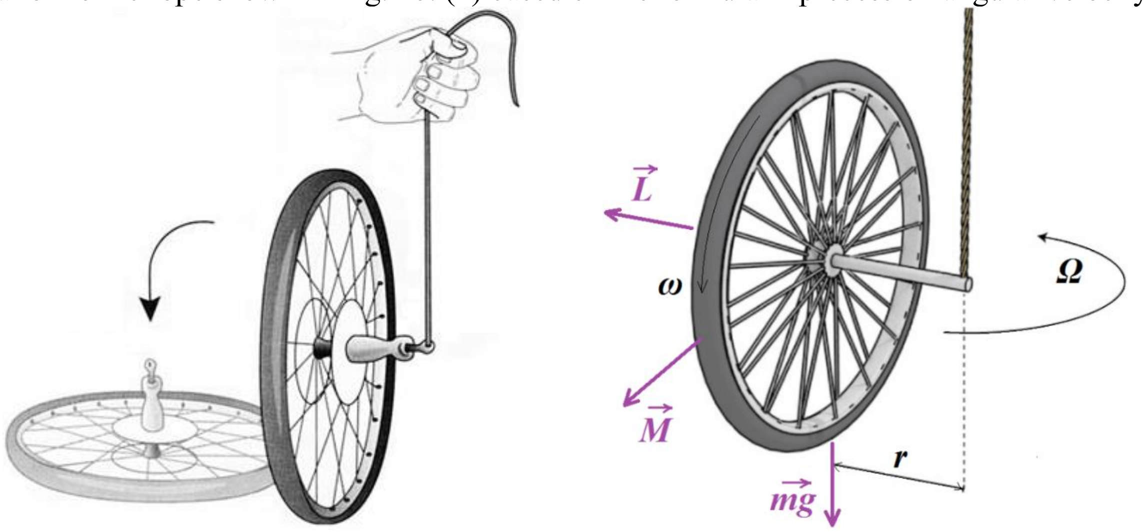
Figure 3. An experimental scheme for the study of precessional movement

*Oh*Angular velocity is called precession angular velocity. According to the formula (15.4), the precession angular velocity is proportional to the magnitude of the external force F. If the external influence F is stopped, then the precession angular velocity Ω immediately becomes zero, that is, it stops rotating as if it were an inertial process. Because we are used to the fact that torque causes the disk to accelerate.

To observe precession clearly, let's look at another case. Let's look at the wheel standing vertically without turning in Fig. 4-a. If we lift it with a rope from one side of the axle of the

Western European Journal of Historical Events and Social Science
Volume 2, Issue 5, May, 2024
https://westerneuropeanstudies.com/index.php/4
ISSN (E): 2942-1926 Open Access| Peer Reviewed

wheel, the wheel will deviate and take a horizontal position as shown in the picture. If the wheel is spinning fast enough, the situation is quite different. Figure 4b shows this situation. In this case, as in the cases discussed in Figure 3, a precessional movement is observed. This precession is caused by a moment of force equal to M=mgr created by the wheel weight mg in relation to the rope shown in Fig. 4b. (4) based on the formula Ω precession angular velocity



a) *b)*

Figure 4. Precessional movement of the wheel

$$\Omega = \frac{mgr}{L} = \frac{mgr}{I\omega} \quad (5)$$

will be If it is assumed that the entire mass of the wheel rests on the flange, then taking the wheel as a mass M of radius R, its moment of inertia $I = mR^2$ it can be said. In this case, the formula (5) is this

$$\Omega = \frac{g\,r}{\omega R^2} \quad (5a)$$

appears. If, instead of a wheel, we take an arbitrary rotating body with a radius of inertia r and a mass m, then formula (5) can be written as follows:

$$\Omega = \frac{g\,r}{\omega \rho^2} \quad (5b)$$

Now let's look at the more general case for the gyroscope. We have observed many times that when we rotate the arrow rapidly in the horizontal plane, the axis of the arrow also rotates around the axis of Oz, forming an angle with the vertical Oz axis. In this case, the lower end A of the elephant almost does not move, and its upper part rotates around the Az axis, forming a precession circle in the horizontal plane (Fig. 5a). To study this situation in detail, we use Figure 5b.

Let the distance from the center of mass of the ball to the base point A be AO=ℓ, the mass of the ball be m, and the moment of inertia about the axis of the ball be equal to I. Let the thunderbolt make an angle α with the vertical axis and rotate rapidly around its core with an

Western European Journal of Historical Events and Social Science
Volume 2, Issue 5, May, 2024
https://westerneuropeanstudies.com/index.php/4
ISSN (E): 2942-1926 Open Access| Peer Reviewed

angular velocity ω. At point A $\vec{Q}$ base reaction force and $\vec{F} = m\vec{g}$ gravity together $\vec{M}$ creates a torque (Fig. 5-b). This is the torque of the couple

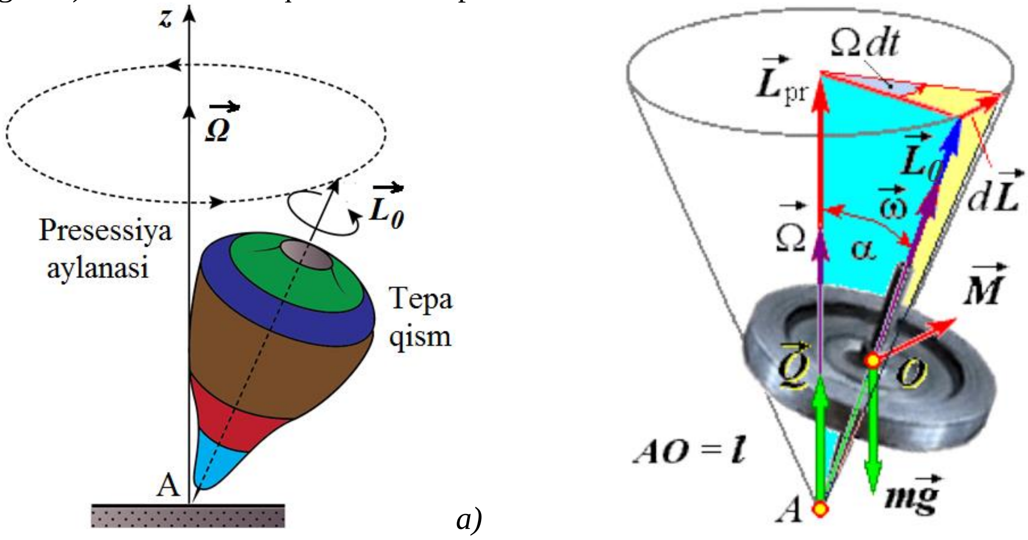


Figure 5. Determining the precession of lightning in general

$$M = mg\ell\sin\alpha \quad (6)$$

will be equal to The increment (or change in angular momentum) that the elemental angular momentum takes in time dt is on the one hand

$$dL = L_0\sin\alpha\, d\theta = (I\omega)\sin\alpha(\Omega dt) \quad (7)$$

is equal to, and on the other hand

$$dL = M\, dt = mg\ell\sin\alpha\, dt \quad (8)$$

will be equal to As a result of equalizing formulas (7) and (8), we create the formula for determining the precession angular velocity Ω.

$$(I\omega)\sin\alpha(\Omega dt) = mg\ell\sin\alpha\, dt, \quad \rightarrow \quad \Omega = \frac{mg\ell}{I\omega}$$

$$\boxed{\Omega = \frac{mg\ell}{I\omega}} \quad (9)$$

The mass m and lengths ℓ in formula (9) can be determined by simple measurements. It is only necessary to determine the moment of inertia I relative to the axis of the shaft and the angular velocity ω given to the shaft. Then it will be possible to calculate the precession angular velocity Ω.

By dividing equation (9) above by equation (8), we can write the formula for the moment of force causing the precession in terms of angular velocities.

$$\boxed{M = I\omega\Omega\sin\alpha} \quad (10)$$

If the gyroscope moves precession around the vertical axis with one end resting on the horizontal plane (a = 900), then the formula (10) takes the following form:

Western European Journal of Historical Events and Social Science
Volume 2, Issue 5, May, 2024
https://westerneuropeanstudies.com/index.php/4
ISSN (E): 2942-1926 Open Access| Peer Reviewed

$$M = I\omega\Omega = L_0\Omega \quad \textbf{(10a)}$$

Now let's solve sample problems related to these studied formulas.

Issue 1. A motorcycle passes a left turn with a radius of R=30 m at a speed of θ=80 km/h. Mass of each wheel m=2.8 kg, outer diameter d=54 cm, radius of inertia r=240 mm. Determine the following: a) moment of inertia of the wheel; b) angular speed of the wheel; c) the precession moment of force resulting from the precession movement of the motorcycle.

Solution: a) We determine the moment of inertia of each wheel.

$$I = m\rho^2 = (2,8kg)\cdot(0,24m)^2 \approx 0,277\ kg\cdot m^2$$

b) We determine the angular speed of rotation of the wheels.

$$\omega = \frac{\vartheta}{r} = \frac{2\vartheta}{d} = \frac{2\cdot\left(\frac{80}{3,6}\frac{m}{s}\right)}{0,54m} \approx 82,3\frac{rad}{s}$$

c) We determine the angular speed of the motorcycle on the turning part of the road.

$$\Omega = \frac{\vartheta}{R} = \frac{\frac{80}{3,6}\frac{m}{s}}{30} = 0,74\frac{rad}{s}$$

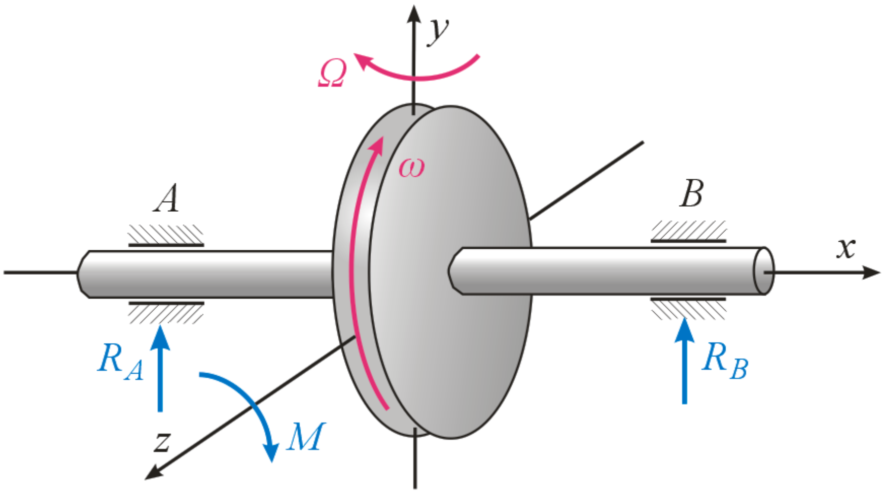
During the movement of the motorcycle in the turn of the road, the axis of its wheels is constantly turning. The angular velocity Ω in the turn can be considered as the precession angular velocity of the free gyroscope. We determine the moment of gyroscopic force due to the precessional movement of the axles of the motorcycle wheels using the formula (10). Note that the number of wheels is 2.

$$M = 2I\omega\Omega = 2\cdot(0,277\,kg\cdot m^2)\cdot\left(82,3\frac{rad}{s}\right)\cdot\left(0,74\frac{rad}{s}\right) = 33,74\ N\cdot m$$

Answer: a) I = 2.77 kg·m2; b) ω = 82.3 rad/s; c) M = 33.74 N·m.

Issue 2. When viewed from the right side, a disc rotating smoothly clockwise with a frequency of n=600 revolutions/minute, mounted on a horizontal shaft, has a radius of inertia of r=70 mm, and a mass of m=5 kg. If this shaft rotates clockwise about a vertical axis with a frequency of np=30 rpm when viewed from above, then determine the reaction forces generated at each bearing due to the weight of the disk and the gyroscopic effect. Bearings A and B are located at a distance of ℓ=120 mm.

Western European Journal of Historical Events and Social Science
Volume 2, Issue 5, May, 2024
https://westerneuropeanstudies.com/index.php/4
ISSN (E): 2942-1926 Open Access| Peer Reviewed



Solution: We determine the angular velocity of the shaft.

$$\omega = 2\pi\nu = 2\pi \cdot \frac{n}{60} = \frac{2\pi \cdot 720 \frac{ayl}{\min}}{60} = 24\pi \frac{rad}{s}$$

We determine the angular velocity in the precession of the shaft.

$$\Omega = 2\pi\nu_p = 2\pi \cdot \frac{n_p}{60} = \frac{2\pi \cdot 30 \frac{ayl}{\min}}{60} = \pi \frac{rad}{s}$$

We determine the moment of inertia of the shaft.

$$I = m\rho^2 = (5kg)\cdot\left(7\cdot 10^{-2} m\right)^2 = 2,45\cdot 10^{-2}\, kg\cdot m^2$$

We determine the moment of gyroscopic force due to the precession movement of the disk axis using formula (10).

$$M = I\omega\Omega = \left(2,45\cdot 10^{-2} kg\cdot m^2\right)\cdot\left(24\pi\frac{rad}{s}\right)\cdot\left(\pi\frac{rad}{s}\right) = 5,8\ N\cdot m$$

This gyroscopic torque vector is directed from our side into the picture plane, and this torque tends to rotate the shaft clockwise from our side. As a result, the gyroscopic torque depresses the right-hand bearing holding the shaft, and lifts the left-hand shaft. The reaction forces in the bearings are in opposite directions according to Newton's 3rd law. In the absence of precession, half the weight of the disk falls on each bearing, but with precession, they are different.

$$R_A = \frac{mg}{2} - \frac{M}{\ell} = \frac{5kg\cdot 9,8\frac{m}{s^2}}{2} - \frac{5,8\,N\cdot m}{0,12\,m} = 24,5\,N - 48,33\,N = -23,83\,N$$

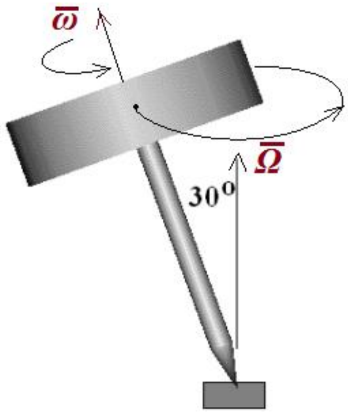
$$R_B = \frac{mg}{2} + \frac{M}{\ell} = \frac{5kg\cdot 9,8\frac{m}{s^2}}{2} + \frac{5,8\,N\cdot m}{0,12\,m} = 24,5\,N + 48,33\,N = 72,83\,N$$

Western European Journal of Historical Events and Social Science
Volume 2, Issue 5, May, 2024
https://westerneuropeanstudies.com/index.php/4
ISSN (E): 2942-1926 Open Access| Peer Reviewed

The fact that the reaction force in bearing A on the left side has a (-) sign can be explained by the fact that the reaction force in this bearing is downward.

Answer:RA = -23.83 N; RB = 72.83N

Issue 3. As shown in the picture, the rotating shaft consists of a rotating disk with a radius of R=40 mm, weight m=0.5 kg, fixed to the end of a lighter rod. The distance from the tip of the blade to the center of the disc is ℓ=100 mm. Calculate the angular velocity in the precessional rotation of a ball rotating around its axis with a frequency of n=30 Hz.



Solution:We determine the angular speed of rotation of the disc around its core.

$$\omega = 2\pi\nu = 2\pi\cdot 30\,Hz = 60\pi\,\frac{rad}{s} = 188,4\frac{rad}{s}$$

We determine the moment of inertia of the disc about its axis of rotation.

$$I = \frac{1}{2}mR^2 = \frac{1}{2}\cdot(0,5kg)\cdot\left(4\cdot 10^{-2}m\right)^2 = 4\cdot 10^{-4}\ kg\cdot m^2$$

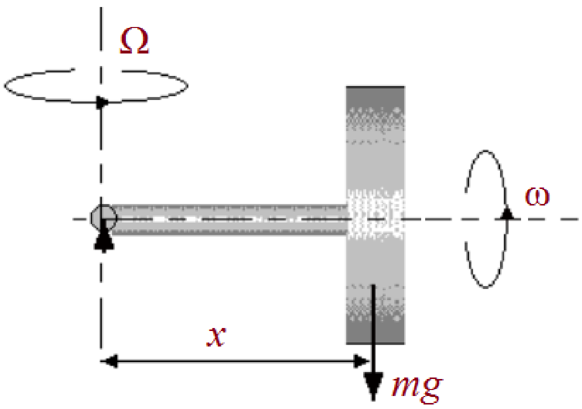
To solve the problem, we use the general formula (9) above.

$$\Omega = \frac{mg\ell}{I\omega} = \frac{(0,5kg)\cdot\left(9,8\frac{m}{s^2}\right)\cdot(0,1m)}{\left(4\cdot 10^{-4}\ kg\cdot m^2\right)\cdot\left(188,4\frac{rad}{s}\right)} = 6,5\,\frac{rad}{s}$$

Answer:Ω = 6.5 rad/s

Now we give several independent assignments for students to solve independently and strengthen their knowledge.

Problem 4. The wheel consists of a rotating disc with a radius R = 50 mm, mass m = 0.8 kg and a light rod attached to the axis of the disc at one end as shown in the figure. The other end of the boom rests horizontally on a fixed support. The distance from the base to the center of the disc is x = 30 mm. If the disc rotates around the core with a frequency n = 30 Hz, then what is the precession angular velocity?



Answer:

Issue 5.A rotating gyroscope has a radius of inertia r = 5 mm. The center of mass of the thunderbolt lies at a distance of d = 30 mm from the point fixed to the ground. At what angular speed ω should the bullet be rotated around its axis so that the angle of deviation of the bolt from the vertical is θ = 200 and the angular velocity of precession is Ω = 0.2 rad/s?**Answer:**58860 rad/s

Problem 6. The ship's motor rotates the tube clockwise as viewed from behind the ship. The moment of inertia of the pipe is I = 250 kg·m2, and it rotates with a frequency of ω = 400 rpm.

Western European Journal of Historical Events and Social Science
Volume 2, Issue 5, May, 2024
https://westerneuropeanstudies.com/index.php/4
ISSN (E): 2942-1926 Open Access| Peer Reviewed

Determine the magnitude and direction of the gyroscopic torque if the bow of the ship plunges down with an angular velocity of $\Omega = 0.2$ rad/s after hitting the wave.**Answer:**209.4 Nm gyroscopic torque deviates from the direction of travel to the left

Problem 7.A monoplane engine is rotating clockwise when viewed from the rear. Moment of inertia of rotating parts I = 300 kg·m2. The motor rotates with a frequency of n = 1200 rpm. The speed of the plane is θ = 1500 km/h, and it is moving in a horizontal plane. From a certain point in time, it started to move to the right along a curve with a radius of R = 5000 m. Determine the magnitude and direction of the resulting gyroscopic moment.**Answer:**M = 3142 Nm of precession torque perpendicular to the aircraft movement; due to the gyroscopic effect, the nose of the aircraft rises and the tail is pushed down, and to balance this, the opposite forces are created with the help of wings.

Thus, we study the most important effect associated with the law of conservation of angular momentum - the precession effect, due to this effect, the precession torque appears when the axis of rotating bodies turns, and we worked out problems related to them . Solving such problems is very useful for future physics teachers, who develop the ability to connect the studied physical laws with situations encountered in technology and life.

References:

1. MBDusmuratov, KTSuyarov, AMTillaboyev. General Physics (Mechanics). - T.: City of books, 2023. 432 p.
2. BXEshchanov, MBDusmuratov, URRustamov. General physics (solving problems in mechanics and molecular physics). - T.: Zebo Print, 2023. 300 p.
3. T. Rashidov, Sh. Shoziyotov, Q.B, Mominov. Theoretical mechanics. Tashkent.: Teacher. 1993.
4. S.Q. Aziz-Qoriev, Sh.Kh. Yangurazov. Solving problems from theoretical mechanics. Tashkent.: Teacher. 1975.
5. D. Giancoli. Physics, part 1. M.: Mir. 1989. 652 st.
6. Ronald J. Hershberger, James J. Reynolds. Calculus with Applications, the 2nd edition.Lexington, Massachusetts.: Copyright © 1993 by DC Heath and Company.
7. Sadhu Singh. Theory of Machines, 3rd edition. - Pearson, New Delhi.: 2012, 264 p.